Robust decentralized multi-channel control system with time delay

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Abstract.
The task is to synthesize a robust multi-channel decentralized control system with delay under the influence of external uncontrollable perturbations. Decentralized robust control for such a system is defined as the task of constructing r local control blocks, for the functioning of which only their measured outputs and inputs are used, while cross-couplings between agents must be compensated. In this case, the required quality of transient processes in local agents of the plant is specified by equations of local reference models. To solve the problem, each agent of the system uses an auxiliary loop method based on the principle of dynamic compensation: a generalized perturbation signal is isolated and then suppressed using an auxiliary loop and two observers with a high gain coefficient. The structure of the synthesized system is such that the isolated perturbation signal contains information not only about the parametric uncertainty of the plant model, but also about the delay and external perturbations acting on each agent individually. The Lyapunov function method is used to prove stability. A numerical example of a system consisting of three agents is considered, illustrating the algorithm's performance for the required dynamic accuracy. The control system is simulated in the Simulink Matlab package. The simulation results confirmed the theoretical conclusions and demonstrated the effectiveness of the control system, taking into account delays in conditions of parametric uncertainty and external uncontrollable perturbations.

.**Keywords:**. robust control, multichannel, decentralized, time delay, auxiliary loop, perturbations.

1 Introduction

The task of compensating for external and internal perturbations acting on a control plant is one of the fundamental tasks in control theory. Solutions to problems under conditions of parameter uncertainty have been obtained by making various assumptions about the control plant model [1-8]. For example, in the case of dynamic deterministic systems, these are assumptions about minimum phase, the order of the relative degree of the model, perturbations acting from outside, etc., and for dynamic stochastic systems, about the Gaussian distribution of measurement errors, the presence of components that generate white noise, etc. An overview of publications on perturbation observers is presented in [1,2].

Control systems that take into account delay components allow for an adequate description of real processes occurring in a dynamic system [8-13]. Thus, publication [10] shows that the exclusion of a small delay in a system of differential equations can lead to a change in the phase portrait. One type of system with delay is state delay, in general, as is known, for the proof of stability of which the apparatus of Lyapunov-Krasovsky functionals is used [9]. In some control problems for plants with time delay, it is possible to obtain a generalized model of the plant and suppress the isolated perturbed signal carrying information about the time delay and perturbations. The auxiliary loop method, based on the principle of dynamic compensation, allows solving perturbations compensation problems, including nonlinear, time-varying, control-delayed, and other components [7,8,12,15-18].

This paper considers the problem of robust compensation for external uncontrollable deterministic perturbations in decentralized control of a multi-channel linear plant with delays in communication channels. The parameters of the plant model are unknown; only the intervals of change in their values are known. Decentralized robust control for such a system is defined as the task of constructing r local control blocks, for the functioning of which only their measured outputs and inputs are used, while cross-connections between agents must be compensated. At the same time, the required quality of transient processes in the local agents of the plant is specified by the equations of local reference models. To solve the problem, each agent of the system uses an auxiliary loop method based on the principle of dynamic compensation: the generalized disturbance signal is isolated and then suppressed using an auxiliary loop and two high-gain observers. This approach eliminates the impact of delays on the control loop and allows for the subsequent solution of the control problem without considering the delay factor. The structure of the synthesized system ensures that the isolated disturbance signal contains information not only about the parametric uncertainty of the plant model, but also about the delay, cross-coupling, and external disturbances affecting each agent individually. The Lyapunov function method was used to prove stability. A numerical example was considered for a linear multichannel plant with time delay under external perturbations and interval uncertainty of the control plant model parameters. System simulation were performed in Simulink packages of the Matlab environment.

2 Problem statement

Consider a multi-channel control plant, whose dynamic processes are described by the following differential equations

$$Q_l(p)y_l(t) = k_l R_l(p)u_l(t) + N_l(p)y_l(t-h_l) + \sum_{j=1, l\neq j}^{r} \Gamma_{lj}(p)y_j(t-h_j) + f_l(t),$$
$$p^i y_l(s) = y_{il}(s), \quad i = 0,\dots,n-1, \;\; s \in [-h_l; 0] \quad l = \overline{1,r}. \qquad (1)$$

where $y_l(t), u_l(t)$ – scalar controlled variables and control actions of local agents, $p = d/dt -$ differentiation operator, $Q_l(p), R_l(p) -$ normalized differential operators, $deg\, Q_l\,(p) = n_l,\ deg\, R_l\,(p) = m_l,\ deg\, N_l\,(p) \le n_l - 1, deg\, \Gamma_{lj}\,(p) \le n_l - 1$, time delay $h_l(t) -$ bounded functions, $k_l > 0,\ f_l(t) -$ external perturbations, $l = \overline{1, r}.$

The reference model for the l subsystem is described by the equation

$$Q_{ml}(p) y_{ml}(t) = k_{ml} g_l(t), \tag{2}$$

where $g_l(t) -$ scalar bounded inputs, $k_{ml} > 0,\ y_{ml}(t) -$ scalar bounded outputs, $deg\, Q_{ml}\,(p) = n_l - m_l, l = \overline{1, r}.$

It is necessary to obtain control algorithms in subsystems that ensure the fulfillment of the following target conditions

$$|y_l(t) - y_{ml}(t)| < \delta_l \text{ при } t \ge T, \tag{3}$$

where δ_l is a sufficiently small number, $T > 0$.

Assumptions.

1. Local subsystems are controllable.
2. The coefficients of operators $Q_l(p), R_l(p), N_l(p), \Gamma_{lj}(p)$ and the values k_l depend on the vector of unknown parameters $\xi_l \in \Xi$, where $\Xi -$ is a known set of possible values of the vector ξ_l, $l = \overline{1, r}.$
3. The delay time in each subsystem $h_l(t) -$ bounded function satisfying the conditions $h_l(t) > 0,\quad l = \overline{1, r}.$
4. The controlling actions $g_l(t)$ and disturbing actions $f_l(t), l = \overline{1, r}$ of local agents are bounded functions of time.
5. Polynomials $R_l(\lambda),\ Q_{ml}(\lambda),\ R_{ml}(\lambda)$ are Hurwitz, where λ is a complex variable in the Laplace transform. $deg\, R_l\,(p) = m_l, deg\, Q_m\,(p) = n_l - m_l, deg\, Q_l\,(p) = n_l$.
6. The derivatives of the output variables and control actions are not measured.

3 Solution

Let us represent operators $Q_l(p)$and $R_l(p)$ as $Q_l(p) = Q_{0l}(p) + \Delta Q_l(p), R_l(p) = R_{0l}(p) + \Delta R_l(p)$, where $Q_{0l}(D), R_{0l}(D) -$ operators with known coefficients such that the polynomials $Q_{0l}(\lambda), R_{0l}(\lambda) -$ are Hurwitz polynomials and have orders n_l and m_l, respectively. Since $Q_{0l}(\lambda)$ and $R_{0l}(\lambda)$ are arbitrary known polynomials, we choose them so that the equality $Q_{ml}(p) = Q_{0l}(p)/R_{0l}(p)$. is satisfied. Then equation (1) takes the form

$$Q_{ml}(p) y_l(t) = k_l \left(u_l(t) + \frac{\Delta R_l(p)}{R_{0l}(p)} u_l(t) - \frac{\Delta Q_l(p)}{k_l R_{0l}(p)} y_l(t) + \frac{N_l(p)}{k_l R_{0l}(p)} y_l(t - h_l(t)) + \frac{\sum_{j=1, l\ne j}^{r} \Gamma_{lj} y_j(t-h_j)}{k_l R_{0l}(p)} + \frac{1}{k_l R_{0l}(p)} f_l(t) \right), l = \overline{1, r}. \tag{4}$$

Let us formulate an equation for the error $e_l(t) = y_l(t) - y_{ml}(t)$, by subtracting (2) from (4),

$$Q_{ml}(p) e_l(t) = k_l \left(u_l(t) + \frac{\Delta R_l(p)}{R_{0l}(p)} u_l(t) - \frac{\Delta Q_l(p)}{k_l R_{0l}(p)} y_l(t) + \frac{N_l(p)}{k_l R_{0l}(p)} y_l(t - h_l) + \frac{\sum_{j=1, l\ne j}^{r} \Gamma_{lj} y_j(t-h_j)}{k_l R_{0l}(p)} + \frac{1}{k_l R_{0l}(p)} f_l(t) - \frac{k_{ml}}{k_l} g_l(t) \right), l = \overline{1, r}. \tag{5}$$

Let us transform equation (5) to the form

$$Q_{ml}(p)e_l(t) = \beta_l u_l(t) + \bar{\varphi}(t),$$

где $\bar{\varphi}(t) = (k_l - \beta_l)u_l(t) + \Big(u_l(t) + \frac{\Delta R_l(p)}{R_{0l}(p)}u_l(t) - \frac{\Delta Q_l(p)}{k_l R_{0l}(p)}y_l(t) + \frac{N_l(p)}{k_l R_{0l}(p)}y_l(t - h_l) + \frac{\sum_{j=1,l\neq j}^{r}\Gamma_{lj}y_j(t-h_j)}{k_l R_{0l}(p)} + \frac{1}{k_l R_{0l}(p)}f_l(t) - \frac{k_{ml}}{k_l}g_l(t)\Big), l = \overline{1,r}.$

We will form a control that will compensate for the negative effect of perturbations in the $l-$ agents. If the measurement $n_l - m_l - 1$ of the derivatives of the control action $v_l(t)$, is available, we will define the control law $u_l(t)$ as

$$u_l(t) = T_l(p)v_l(t), l = \overline{1,r}. \tag{6}$$

Then equation (5) will take the following form

$$Q_{ml}(p)e_l(t) = k_l T_l(p)(v_l(t) + \frac{\Delta R_l(p)}{R_{0l}(p)}v_l(t) - \frac{\Delta Q_l(p)}{k_l R_{0l}(p)T_l(p)}y_l(t) + \\ + \frac{N_l(p)}{k_l R_{0l}(p)T_l(p)}y_l(t - h_l(t)) + \frac{\sum_{j=1,l\neq j}^{r}\Gamma_{lj}y_j\big(t-h_j\big)}{k_l R_{0l}(p)T_l(p)} \\ + \frac{1}{k_l R_{0l}(p)T_l(p)}f_l(t) - \frac{k_{ml}}{k_l T_l(p)}g_l(t)), \quad l = \overline{1,r}. \tag{7}$$

Since the system is designed on the assumption that only scalar inputs and outputs are available for measurement, the control law will be defined instead of (6) as

$$u_l(t) = T_l(p)\overline{v_l}(t), l = \overline{1,r}. \tag{8}$$

where $\overline{v}_l(t)$ is the signal estimate obtained from the observer [14]

$$\dot{\xi}_l = F_{0l}\xi_l(t) + B_{0l}(v_l(t) - \overline{v}_l(t)), \\ \overline{v}_l(t) = L_l\xi_l(t), l = \overline{1,r}. \tag{9}$$

Here $\xi_l(t) \in R^{n_l-m_l}, F_{0l}$ is a Frobenius matrix with a zero bottom row, $L_l = [1,0,\ldots,0], B_{0l}^T = \left[\frac{b_{1l}}{\mu},\ldots,\frac{b_{n_l-m_l}}{\mu^{n_l-m_l}}\right]$. The parameters $b_1,\ldots,b_{n_l-m_l}$ are chosen so that the matrices $F_l = F_{0l} + B_l L$ are Hurwitz, ${B_l}^T = [b_{1l},\ldots,b_{n_l-m_l}]$.

Substituting (8) into (7), we obtain the equation

$$Q_{ml}(p)e_l(t) = \beta_l T_l(p)v_l(t) + \overline{\phi_l}(t) + \beta_l T_l(p)(\overline{v}_l(t) - v_l(t)), \tag{10}$$

where

$$\overline{\phi}_l(t) = (k_l - \beta_l)v_l(t) + k_l T_l(p)(\frac{\Delta R_l(p)}{R_{0l}(p)}v_l(t) - \frac{\Delta Q_l(p)}{k_l R_{0l}(p)T_l(p)}y_l(t) + \\ + \frac{N_l(p)}{k_l R_{0l}(p)T_l(p)}y_l\big(t - h_l(t)\big) + \frac{\sum_{j=1,l\neq j}^{r}\Gamma_{lj}y_j\big(t-h_j\big)}{k_l R_{0l}(p)T_l(p)} + \\ + \frac{1}{k_l R_{0l}(p)T_l(p)}f_l(t) - \frac{k_{ml}}{k_l T_l(p)}g_l(t)).$$

Let us select a polynomial $T_l(\lambda)$ in the $l-th$ local agent so that the transfer function satisfies the equality $\frac{T_l(\lambda)}{Q_{ml}(\lambda)} = \frac{1}{\lambda + a_{ml}}$. Then equation (10) is transformed to the form

$$(p + a_{ml})e_l(t) = \beta_l v_l(t) + \phi_l(t), \tag{11}$$

where $\phi_l(t) = \frac{1}{T_l(p)}\overline{\phi}_l(t) + \beta_l(\overline{v}_l(t) - v_l(t)), l = \overline{1,r}.$

The signal $\phi_l(t)$ concentrates all the uncertainty of the parameters of the $l-th$ local control agent, information about external perturbations, nonlinearities, and delays.

To compensate for the negative effect of the selected signal $\phi_l(t)$ in the в $l-th$ local agent, we will use the auxiliary loop method [7].

Let's introduce an auxiliary loop

$$(p+a_{ml})\overline{e}_l(t)=\beta_l v_l(t), l=\overline{1,r}. \tag{12}$$

Taking into account (11) and (12), we can formulate the equation

$$(p+a_{ml})\zeta_l(t)=\phi_l(t),\ l=\overline{1,r}, \tag{13}$$

where $\zeta_l(t)=y_l(t)-\overline{y}_l(t), \zeta_l(t)-$ discrepancy. Thus, if n_l-m_l-1 derivatives of signals $v_l(t)$ and the first derivative of the controlled variable $e_l(t)$, are available for measurement, then, by forming $v_l(t)$ in the form

$$v_l(t)=-\frac{1}{\beta_l}(p+a_{ml})\zeta_l(t), l=\overline{1,r}, \tag{14}$$

we obtain that the control law (6), (14) ensures asymptotic stability of the system (1), (6), (14) with respect to the variable e_l (t), and the equation of the closed system will be of the form $(p+a_{ml})e_l(t)=0$. Therefore, from (5) we have that n_l-m_l derivatives of the signal $e_l(t)$ tend to 0. But for the system to be operational, it is necessary to show that the signal $\phi_l(t), l=\overline{1,r}$ is bounded.

So, if it is possible to measure the listed derivatives $\phi_l(t)=\frac{1}{T_l(P)}\overline{\phi}_l(t)$, and from (14) we have that $v_l(t)=-\frac{1}{\beta_l}\phi_l(t)$. Then

$$u_l(t)=-\frac{1}{\beta_l}((k_l-\beta_l)u_l(t)+k_l\frac{\Delta R_l(p)}{R_{0l}(p)}u_l(t)-\frac{\Delta Q_l(p)}{R_{0l}(p)}y_l(t)+$$
$$+\frac{N_l(p)}{R_{0l}(p)}y_l(t-h_l(t))+\frac{1}{R_{0l}(p)}f_l(t)-\frac{k_{ml}}{R_{0l}(p)}g_l(t)\Big) \ . \tag{15}$$

The components $\frac{\Delta Q_l(p)}{R_{0l}(p)}y_l(t), \frac{N_l(p)}{R_{0l}(p)}y_l(t-h_l(t)), \frac{1}{R_{0l}(p)}f_l(t)$ are bounded due to the Hurwitz property of the polynomial $R_{0l}(\lambda)$ and the condition $\lim_{t\to\infty} e_l(t)=0$. Furthermore, the control action $g(t)$ is bounded due to assumption.

Solving equation (15) $u_l(t)$, for we obtain

$$u_l(t)=\Big(-\frac{\Delta R_l(p)}{R_{0l}(p)}u_l(t)+\frac{\Delta Q_l(p)}{k_l R_{0l}(p)}y_l(t)-\frac{N_l(p)}{k_l R_{0l}(p)}y_l(t-h_l(t))-$$
$$-\frac{1}{k_l R_{0l}(p)}f_l(t)+\frac{k_{ml}}{k_l R_{0l}(p)}g_l(t)\Big). \tag{16}$$

From which, taking into account (4), it follows that $Q_{ml}(p)y_l(t)=0$, and from (15) we have

$$u_l(t)=\frac{-R_{0l}(p)}{R_{0l}(p)+\Delta R_l(p)}=\Big(\frac{\Delta Q_l()}{kR_{0l}(p)}y_l(t)-\frac{N_l(p)}{k_l R_{0l}(p)}y_l(t-h_l(t))-$$
$$-\frac{1}{k_l R_{0l}(p)}f_l(t)+\frac{k_{ml}}{k_l R_{0l}(p)}g_l(t)\Big).$$

Since $R_{0l}(p)+\Delta R_l(p)=R_l(p)$ is a Hurwitz polynomial, and n_l-m_l derivatives $e_l(t)$ tend to zero, the control signal $u_l(t)$, is bounded, and therefore the signals $\phi_l(t), \overline{\phi}_l(t)$, variable $\zeta_l(t)$ and its derivative due to (14).

If it is impossible to measure the necessary derivatives of the signal $\zeta_l(t)$, instead of (14), we form the signal $v_l(t)$ in the form

$$v_l(t) = -\frac{1}{\beta_l}(p + a_{ml})\overline{\zeta_l}(t), l = \overline{1,r}, \quad (17)$$

where $\overline{\zeta}_l(t)$ − assessment obtained from an observer [14]

$$\dot{z}_l = \overline{F_{0l}} z_l(t) + \overline{B_{0l}}(\zeta_l(t) - \overline{\zeta}_l(t)),$$
$$\overline{\zeta}_l(t) = L_{l2} z_l(t), \quad l = \overline{1,r}, \quad (18)$$

where $z_l(t) \in R^2$, matrices F_{0l} and B_{0l} are the same as in (9), but only of the corresponding dimensions, $L_2 = [1{,}0]$, $l = \overline{1,r}$. In this case, the statement is true.

Theorem. Let assumptions 1-6 hold, then for any $\delta > 0$ in (3) there exist numbers such that when $\mu_l > 0,\ T > 0$ and $t \geq T$ for systems (1), (8), (9), (12), (17), (18), the target conditions (3) are satisfied and all variables in the system are bounded.

Proof. Let us introduce the vectors

$$\sigma_l^T(t) = (v_l(t), pv_l(t), \ldots, pv_l(t)),\ z_{0l}^T = [\zeta_l(t), p\zeta_l(t)]$$

and normalized discrepancy vectors

$$\overline{\eta}_l(t) = \Gamma_{1l}^{-1}(\sigma_l(t) - \xi_l(t)),\ \overline{w}_l(t) = \Gamma_{2l}^{-1}(z_{0l}(t) - z_l(t)).$$

Here $\Gamma_{1l}^{-1} = diag\{{\mu_l}^{n_l - m_l - 1}, \ldots, \mu_l, 1\}$, $\Gamma_{2l}^{-1} = diag\{\mu_l, 1\}$. Then, from (9) and (18), we have

$$\begin{cases} \dot{\overline{\eta_l}}(t) = \frac{1}{\mu_l} F_l \overline{\eta}_l - b_{0l} p^{n_l - m_l} v_l(t), \\ \theta_l(t) = {\mu_l}^{n_l - m_l - 1} L_l \overline{\eta_l}(t), l = \overline{1,r}, \end{cases} \quad \begin{cases} \dot{\overline{w}}_l(t) = \frac{1}{\mu_l} \overline{F}_l \overline{w}_l(t) - p^2 \zeta_l(t), \\ \tau_l(t) = \mu_l L_{2l} \overline{w}_l(t), \quad l = \overline{1,r}, \end{cases} \quad (19)$$

где $\overline{F_l} = \overline{F_{0l}} + \overline{B_{0l}} L_{l2}$, $b_{l0}^T = [0, \ldots, 1]$, $\overline{b_{0l}^T} = [0{,}1]$, $L_{2l} = [1{,}0]$, $\theta_l(t) = v_l(t) - \overline{v_l}(t)$, $\tau_l(t) = \zeta_l(t) - \overline{\zeta_l}(t)$, $l = \overline{1,r}$.

Let us transform equations (19) into equivalent equations with respect to outputs $\theta_l(t)$ and $\tau_l(t)$

$$\begin{cases} \dot{\eta}_l(t) = \frac{1}{\mu_l} F_l \eta_l(t) - bpv_l(t), \\ \theta_l(t) = {\mu_l}^{n_l - m_l - 1} L_l \eta_l(t), \end{cases} \begin{cases} \dot{w}_l(t) = \frac{1}{\mu_l} \overline{F}_l w_l(t) - \overline{b_l} p \zeta_l(t) \\ \tau_l(t) = \mu_l L_{2l} w_l(t), \end{cases} \quad (20)$$

$b_l^T = [1{,}0, \ldots, 0]$, $\overline{b_l}^T = [1{,}0]$, $l = \overline{1,r}$.

Equations (19) and (20) are equivalent with respect to outputs $\theta_l(t)$ and $\tau_l(t)$, since they are vector-matrix forms of the same equations:

$$\left(p^{n-m} + \frac{b_{l1}}{\mu_l} p^{n_l - m_l - 1} + \ldots + \frac{b_{n_l - m_l}}{{\mu_l}^{n_l - m_l}}\right) \theta_l(t) = p^{n_l - m_l} v_l(t),$$
$$\left(p + \frac{d_{1l}}{\mu_l} p + \frac{d_{2l}}{\mu_{l2}}\right) \tau_l(t) = p^2 \zeta_l(t), \quad l = \overline{1,r}.$$

Taking into account (6), (17), and (20), equation (11) will take the form

$$e_l(t) = -\mu_l L_2 w_l(t), \quad l = \overline{1,r}. \quad (21)$$

Let us take the Lyapunov function

$$V_l(t) = {\eta_l}^T(t) H_l \eta_l(t) + {w_l}^T(t) H_{1l} w_l(t), \quad l = \overline{1,r},$$

where the positive definite matrices H_l and H_{1l} are solutions to the equations

$$H_l F_l + {F_l}^T H_l = -2\rho_{1l} I,\ H_{1l} \overline{F_l} + \overline{F_l}^T H_{1l} = -2\rho_{2l} I_l, \quad l = \overline{1,r},$$

and calculate the total derivative on the trajectories of the system

$$\dot{V}_l(t) = -2\frac{\rho_{1l}}{\mu_l} |\eta_l(t)|^2 - 2\frac{\rho_{2l}}{\mu_l} |w_l(t)|^2 - 2{\eta_l}^T(t) H_l pv_l(t) - {w_l}^T(t) H_{1l} p_l \zeta_l(t). \quad (22)$$

Let us write equations (20) in the form

$$\begin{cases} \mu_{1l}\dot{\eta}_l(t) = F_l\eta_l(t) - \mu_{2l}b_l p v_l(t), \\ \mu_{1l}\dot{w}_l(t) = \overline{F_l}w_l(t) - \mu_{2l}\overline{b}_l p\zeta_l, \\ (p + a_{ml})y_l = -\mu_{2l}(p + a_{ml})L_{2l}w_l(t), \\ \mu_{1l} = \mu_{2l}, \quad l = \overline{1,r} \end{cases} \tag{23}$$

Let us use Lemma [7].

Lemma [7]. If the system is described by the equations

$$\dot{x} = f(x, \mu_1, \mu_2), \quad x \in R^n,$$

$\mu = col(\mu_1, \mu_2)$, $f(x, \mu_1, \mu_2) -$ is a continuous function, Lipschitzian with respect to x, and when $\mu_2 = 0$ has a bounded closed region of dissipativity $\Omega = \{x: F(x) < C\}$, where $F(x)$ is a positive definite, continuous, piecewise smooth function, then there exists $\mu_0 > 0$ such that when $\mu_1 < \mu_0$ и $\mu_2 < \mu_0$, the initial system has the same dissipativity region Ω, if for some numbers $C > 0, \overline{\mu}_1 > 0,\ \mu_2 = 0$ the condition

$$\sup_{|\mu_1| \le \overline{\mu_1}} \left(\left(\frac{\partial F(x)}{\partial x} \right)^T f(x, \mu_1, 0) \right) \le -C,\ F(x) = C$$

is satisfied.

In this case, if $\mu_{l2} = 0$ in (23), then this is equivalent to all derivatives being measured and two exponentially stable systems being added

$$\mu_{l1}\dot{\eta}_l(t) = F_l\eta_l(t),\ \dot{w}_l(t) = \overline{F}_l w_l(t).$$

As already proven, in this case all variables in the system are bounded and the conditions of the lemma are satisfied. In other words, in the region Ω $e_l(t) \to 0$, $|v_l(t)| < k_{1l}$, $|\zeta_l(t)| < k_{2l}$, and from (8) and (15) it follows that $|p\zeta_l(t)| < k_{3l}$, $|pv_l| < k_{4l}$, where $k_{1l}, k_{2l}, k_{3l}, k_{4l} -$ are some positive constants. Let us show that in the case of $\mu_{2l} \neq 0$or more precisely, when$\mu_{1l} < \mu_{0l}$ и $\mu_{2l} < \mu_{1l}$, the region of dissipativity will be . Ω_l.

Let us $\mu_{1l} = \mu_{2l} = \mu_{l_l}$ in (23) and, substituting them into (22), use the estimates

$$-2\eta_l^T(t)H_l p v_l(t) \le \frac{1}{\mu_l}|\eta_l(t)|^2 + \mu_l\|H_l\|^2|pv_l(t)|^2 \le \frac{1}{\mu_l}|\eta_l(t)|^2 + \mu_l\|H_l\|^2 k_{3l}^2,$$

$$-2w_l^T(t)H_{1l}p\zeta_l(t) \le \frac{1}{\mu_l}|w_l(t)|^2 + \mu_l\|H_{1l}\|^2 k_{4l}^2, \quad l = \overline{1,r}.$$

Substituting these estimates into (22), we obtain the inequality

$$\dot{V}_l(t) \le -\frac{\rho_{1l}}{\mu_l}|\eta_l(t)|^2 - \frac{\rho_{2l}}{\mu_l}|w_l(t)|^2 - \frac{1}{\mu_l}(\rho_{1l} - 1)|\eta_l(t)|^2 - \frac{1}{\mu_l}(\rho_{2l} - 1)|w_l(t)|^2 +$$
$$+\mu_l(\|H_l\|^2 k_{3l}^2 + \|H_{1l}\|^2 k_{4l}^2), \quad l = \overline{1,r}.$$

By choosing $\rho_{1l} > 1$ и $\rho_{2l} > 1$, we obtain

$$\dot{V}_l(t) \le -\frac{\rho_{1l}}{\mu_l}|\eta_l(t)|^2 - \frac{\rho_{2l}}{\mu_l}|w_l(t)|^2 + \mu_l\beta_{3l}, \tag{24}$$

where $\beta_{3l} = \|H_l\|^2 k_{3l}^2 + \|H_{1l}\|^2 k_{4l}^2$. Where should one go

$$\dot{V}_l(t) \le -\beta_{1l}V_l(t) + \mu_l\beta_{3l}, \qquad (25)$$

where $\beta_{l1} = min\left\{\frac{\rho_{1l}}{\mu_l\overline{\lambda}(H_l)}; \frac{\rho_{2l}}{\mu_l\overline{\lambda}(H_{1l})}\right\}$, $\overline{\lambda}(\cdot)$ – the maximum eigenvalue of the corresponding matrix. From (25) we have $V_l(t) \le \frac{\mu_l\beta_{3l}}{\beta_{1l}}$.

Taking into account the inequality $|w_l(t)|^2 \le \frac{1}{\overline{\lambda_l}(H_{1l})}V_l(t) \le \frac{\mu_l\beta_l}{\beta_{1l}}$ из from the third equation (23), we have $|e_l(t)| \le \mu_l|w_l(t)| \le \mu_l\sqrt{\frac{\mu_l\beta_{3l}}{\beta_{1l}}}$. From this, we can see that for any $\delta_l > 0$ in (3), there exists μ_{0l} such that the target condition (3) will be satisfied.

4 Numerical example

Consider a multi-chanal plant consisting of three control agents, whose dynamic processes are described by differential equations

$$(p^4 + a_{1l}p^3 + a_{2l}p^2 + a_{3l}p + a_{4l})y_{1l}(t) = (b_{0l}p + b_{1l})u_1 + (c_{1l}p + c_{2l})f_l(t) +$$
$$+(n_{1l}p^3 + n_{2l}p^2 + n_{3l}p + n_{4l})y_j(t - h_l), \quad l = \overline{1,3}, j = \overline{1,3}, \ j \ne l.$$

The task of decentralized agent control, as noted in assumption 2, is solved under conditions of parametric uncertainty.

The uncertainty class is defined by the following inequalities:

: $-2 \le a_{ql} \le 8$, $1 \le b_{0l} \le 20$, $1 \le b_{1l} \le 20$, $-1 \le c_{1l} \le 15$, $-1 \le c_{2l} \le 15$, $-10 \le n_{ql} \le 10, q = \overline{1,4}, l = \overline{1,3}$.

The equations of the reference agent models are as follows:

$$(p+3)^3 y_{m1}(t) = 15g_1(t)),$$
$$(p+2)^3 y_{m2}(t) = 10g_2(t),$$
$$(p+4)^3 y_{m3}(t) = 12g_3(t),$$
$$g_1(t) = sin1.7t, g_2(t) = sin2t, g_3(t) = sin4t.$$

Following the control algorithms proposed in this paper, we select:

- polynomials $T_1(\lambda) = (\lambda+3)^2$ $T_2(\lambda) = (\lambda+2)^2, T_1(\lambda) = (\lambda+4)^2$, $\beta_l = 20, \mu_l = 0{,}01, l = \overline{1,3}$.
- auxiliary loop in the form of

$$(p+3)\overline{e}_1(t) = 20v_1(t),$$
$$(p+2)\overline{e}_2(t) = 20v_2(t),$$
$$(p+4)\overline{e}_3(t) = 20v_3(t);$$

- observer equations in the form

$$\begin{cases} \dot{\xi}_{l1}(t) = \xi_{l2}(t) + \frac{6}{\mu_l}\big(v_l(t) - \xi_{l1}(t)\big), \\ \dot{\xi}_{l2}(t) = \frac{8}{\mu_l^2}\big(v_l(t) - \xi_{l1}(t)\big) \\ \overline{v}_l(t) = \xi_{l1}(t), l = \overline{1,3}, \end{cases}$$

$$\begin{cases} \dot{z}_l(t) = \frac{3}{\mu_l}(\zeta_l(t) - z_l(t)), \\ \overline{\zeta}_l(t) = z_l(t), \quad l = \overline{1,3}. \end{cases}$$

- control actions in the form of

$$u_1(t) = \xi_{11}(t) + 6\xi_{21}(t) + 9\dot{\xi}_{21}(t),$$
$$u_2(t) = \xi_{12}(t) + 4\xi_{22}(t) + 4\dot{\xi}_{22}(t),$$
$$u_3(t) = \xi_{13}(t) + 8(t) + 16\dot{\xi}_{21}(t),$$

$$v_1(t) = -0.05\big(3\zeta_1(t) + \dot{z}_1(t)\big),$$
$$v_2(t) = -0.05\big(2\zeta_2(t) + \dot{z}_2(t)\big),$$
$$v_3(t) = -0.05(4\zeta_3(t) + \dot{z}_3(t)).$$

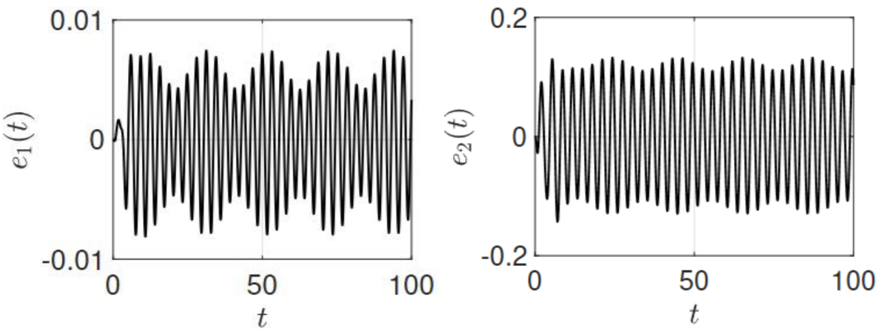
The proposed control algorithms were simulated in the Simulink package of the Matlab environment for a plant of three control agents, whose mathematical models are represented as follows:

$$(p^4 - 4p^3 + 4p^2 - 4p + 4)y_1(t) = (7p + 5)u_1 + (3p + 1)f_1(t) +$$
$$+(3p^2 + 3p + 3)y_1(t - 2) + (p^3 + 2p^2 + p + 1)(y_2(t - 2) + y_3(t - 2)),$$

$$(p^4 - 2p^3 + 8p^2 - p - 2)y_2(t) = (p + 5)u_2 + (p + 1)f_2(t) +$$
$$+(-p + 2)y_2(t - 2) + (5p^3 + 2p^2 + 3p + 1)(y_1(t - 2) + y_3(t - 2)),$$

$$(p^4 - 4p^3 + 4p^2 - p + 1)y_3(t) = (3p + 1)u_3 + (p + 4)f_3(t) +$$
$$+(4p^2 + p + 1)y_3(t - 2) + (p^3 + 2p^2 - p + 1)(y_1(t - 2) + y_2(t - 2)).$$

Fig. 1 shows the tracking errors $e_1(t), e_2(t), e_3(t)$ trajectories.



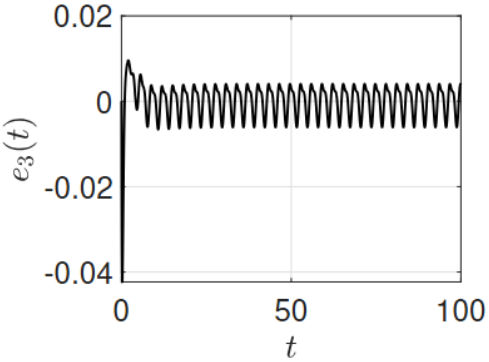


Fig. 1. Tracking error trajectories

Fig. 2 shows the tracking the trajectories of reference models $y_{m1}(t), y_{m2}(t), y_{m3}(t)$.

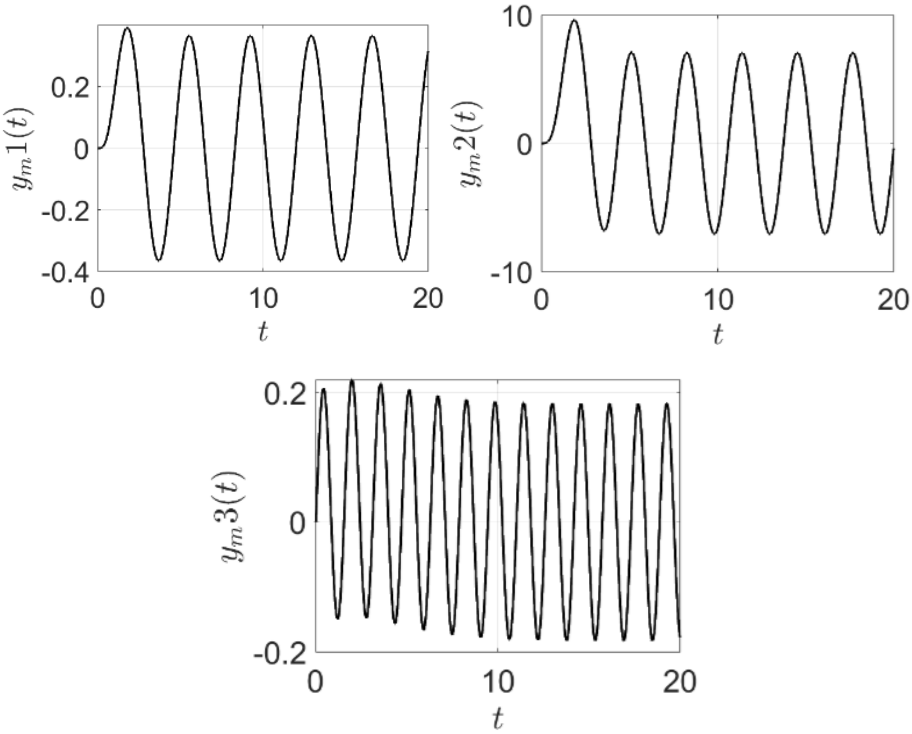


Fig. 2. Tracking the trajectories of reference models

Fig. 3 shows the tracking the trajectories of control actions

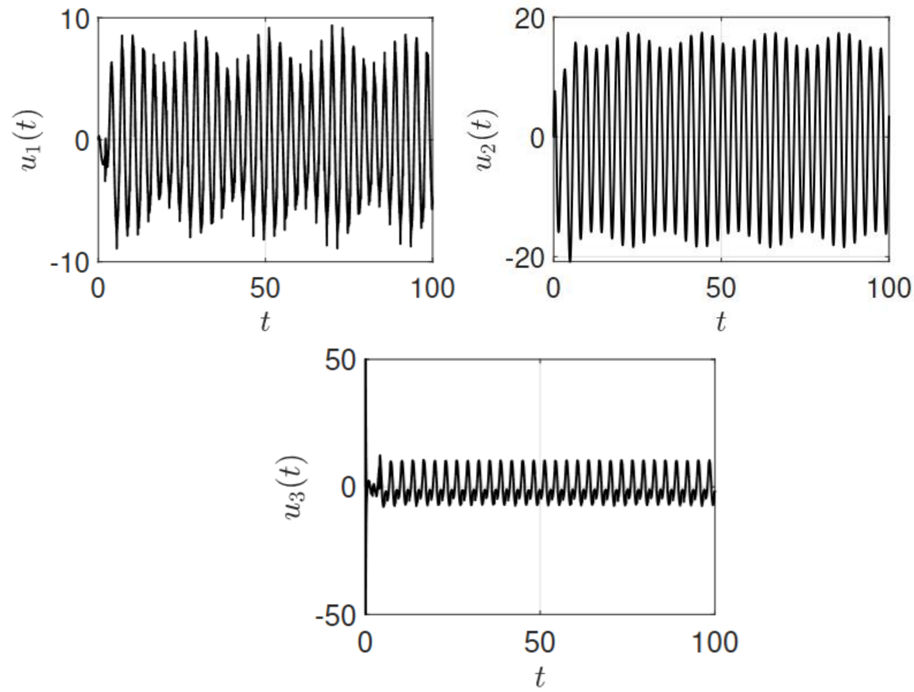


Fig. 3. Tracking the trajectories of control actions

The computer simulation results shown in the figures demonstrate the good performance of a decentralized multi-channel system that takes into account delays, cross-signals, external disturbances, and a priori uncertainty in the parameters of the plant agents.

5 Conclusion

The paper proposes a novel robust decentralized control system for a multi-channel linear plant with delay. Each agent of the plant compensates for external and internal disturbances using the auxiliary loop method. Based on the estimates of disturbances, control actions are generated to achieve the control goal with a specified accuracy. The structure of the synthesized system is such that the isolated disturbance signal contains information not only about the parametric uncertainty of the plant model, but also about the time delay, cross-connections, and external disturbances affecting each agent individually. The Lyapunov function method was used to prove stability. A numerical example was considered for a linear multi-channel system with time delay under external disturbances and interval uncertainty in the control system. The results of computer modeling in the Simulink package of the Matlab environment confirmed the theoretical findings and demonstrated good performance of the control system. This approach can be applied and extended to other multi-channel control plants, in particular, agents that are described by nonlinear, non-stationary differential equations with and without delay, as well as for the case of non-identical agents with equal relative degrees of agent models.

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