Adaptive Kalman Filter with Recursive System Noise Covariance Estimation Applied to UAV Dynamics

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Abstract— **A novel covariance difference-based system noise covariance estimation method for Kalman filter tuning is proposed. Q- adaptive Kalman filter with recursive system noise covariance estimation is derived. Influence of the system noise bias to the state correction sequence of Kalman filter (KF) is investigated. It is shown in the study that the bias type system noise change may be converted to the mean square of state correction sequence of KF and such type of changes can be compensatd using the covarince matching techniques. The proposed adaptive adjustment of system noise covariance in Kalman filter is applied for the model of unmanned aerial vehicle (UAV) dynamics. Algorithm is tested for different system noise bias scenarios. The simulation results show that the proposed covariance difference-approach based adaptive Kalman filter (AKF) with recursive Q-adaptation can estimate the UAV dynamics accurately in real time in the presence of system noise uncertainties. The estimation accuracies of the adaptive and non-adaptive versions of the Kalman filter are compared in the presence of system noise bias type changes.**

Key words: kalman filter, state correction sequence, adaptive estimation, unmanned aerial vehicle, system noise, covariance estimation

I. Introduction

One of the difficulties with conventional Kalman filters is tuning the filter with respect to the process noise covariance. Changes in disturbances, unmanned aerial vehicles (UAVs) dynamics modeling errors, and any actuator problems cause the filter's process noise covariance to change. This type of process noise covariance is too variable for the conventional Kalman filter to adapt to. The extended Kalman filter (EKF), uncented Kalman filter (UKF), and other conventional Kalman filtering methods all have the same drawback when used for state estimation: they are not robust to changes in the environment. The accuracy of the filter estimate degrades as these changes occur, and the filter may diverge if faults persist over a long period of time. As a result, we seek an adaptive Kalman filter (AKF) method that can withstand changes in the statistical characteristics of the system noise.

Several techniques for suggesting adaptive Kalman filter extensions have been proposed [1–3]. One of these ideas is scaling the covariance of the process noise. The scaling parameters can be computed in a variety of ways [3–7].

An adaptive Kalman filter for orientation estimation in accelerated and ferromagnetic disturbed situations was introduced in [5]. It is based on the adaptive factor and the Chi-square test. To best compensate for the erroneous process noise covariance matrices and the unmodeled gyroscope biases, an adaptive component was added to the process noise covariance matrix. The suggested approach, which was based on the adaptive Kalman filter, could accurately estimate orientation and external disturbances in a variety of scenarios.

In [6], an improved adaptive unscented Kalman filter is presented. The proposed method can be used to estimate unknown input data and the diagonal process noise covariance matrix simultaneously. In this study, the adaptive KF for covariance matrix estimation is combined with KF estimators for unknown input data.

The paper [8] proposes using multiple fading factors to adjust the process noise covariance matrix by giving more weight to sensor measurements than corrupted state predictions caused by model uncertainties. In [9] and [10], two noise covariance matching criteria are created and combined: an innovation-based method for process noise covariance and a residual-based method for measurement noise covariance, but the fault type (process or measurement) cannot be determined in this situation.

In [4], an adaptation strategy is proposed to adjust the process noise covariance matrix (Q) based on only one fading element. The differences in the effects of changing the process noise covariance on the estimation performance of each estimated state are the main reason for adopting multiple attenuation factors. The influence of the process noise covariance change on each state should be carefully examined, especially for complicated multivariable systems, and instead of using a single component, a matrix made up of many fading factors should be utilized (such that it weights the adaptation differently for each state). The suitable Q-adaptive filter based on multiple fading factors is proposed in [7].

In this study a covariance difference-based adaptive Kalman filter with recursive Q-adaptation is presented. It is shown that the system noise bias type system changes will cause a change in the statistical characteristics of the state correction sequence of Kalman filter. For the purpose of estimating the UAV states, simulations are compared using the adaptive and non-adaptive versions of the Kalman filters in the presence of system noise bias type system changes.

II. Problem Statement

Let us consider the linear dynamic system described by the state equation

$$x(k+1) = Ax(k) + Bu(k) + w(k) \quad (1)$$

and measurement equation

$$z(k) = H(k)x(k) + V(k) \quad (2)$$

where $x(k)$ is the vector of system state; A is the transition matrix of the system; B is the control distribution matrix; $u(k)$ is the control input vector; $w(k)$ is the random vector of disturbances (system noise); $z(k)$ is the vector of measurements; $H(k)$ is the matrix of measurements of the system; and $V(k)$ is the random vector of measurement noise. Assume that random vectors $w(k)$ and $V(k)$ are mutually uncorrelated Gaussian white noise with zero mean and covariances

$$\begin{aligned} &E\left[w(k)w^T(j)\right] = Q(k)\delta(kj); \\ &E\left[V(k)V^T(j)\right] = R(k)\delta(kj). \end{aligned} \quad (3)$$

According to [11], the linear optimal Kalman filter (OKF) estimating the state vector of system (1) is formulated using the following recursive equations:

Equation of the prediction value,

$$\hat{x}(k/k-1) = A\hat{x}(k-1/k-1) + Bu(k-1) \quad (4)$$

Equation of the estimation value,

$$\hat{x}(k/k) = \hat{x}(k/k-1) + K(k)\left[z(k) - H(k)\hat{x}(k/k-1)\right] \quad (5)$$

where $K(k)$ is the gain matrix of the OKF.

The state correction sequence,

$$\varepsilon(k) = \hat{x}(k/k) - \hat{x}(k/k-1) \quad (6)$$

The state correction covariance

$$P_\varepsilon(k) = P(k/k-1) - P(k/k) \quad (7)$$

where $P(k/k-1)$ and $P(k/k)$ are the covariances of the prediction and estimation errors, respectively. However, in practice, the above assumptions (3) may not be satisfied, i.e., the mean values of system and measurement noise, as well as covariances, are not constant and may change during the operation of the system. This study requires investigating the effect of system noise bias on the estimation properties of the Kalman filter and designing a Kalman filter that is robust to changes in the mean value of the system noise.

III. Covariance Matching-Based Q-Adaptive AKF

A. *Influence of system noise bias to the Kalman filter estimation*

Assumpton. In this section, the system noise is assumed to be biased and can be represented as sum of random w(k) and constant $\lambda(k)$ components.

The following theorem is proved in this study.

Theorem: If the measurements are processed by the OKF (4)-(7) and a system noise bias arises at an iteration step $k = \tau$, then at all $k \geq \tau$ steps the prediction, estimation and state correction of OKF will be biased.

B. *Development of Q-Adaptive AKF*

The difference between the estimation and extrapolation values (state correction sequence) (6) is proposed to use for the system noise covariance estimation purpose. In [12], it is proved that the state correction sequence (the difference between the state before and after updates)

$$\begin{aligned} &\varepsilon(k) = \hat{x}(k/k) - \hat{x}(k/k-1) = \\ &\hat{x}(k/k) - A\hat{x}(k-1/k-1) + Bu(k-1) \end{aligned} \quad (8)$$

is Gaussian with zero mean and covariance

$$P_\varepsilon(k) = E\left[\varepsilon(k)\varepsilon^T(k)\right] = P(k/k-1) - P(k/k) \quad (9)$$

The system noise covariance matching procedure is mainly aimed at finding an appropriate system noise covariance matrix $Q(k)$ such that the real and theoretical values of the state correction sequence covariance match

$$\frac{1}{M}\sum_{j=k-M+1}^{k} \varepsilon(j)\varepsilon^T(j) = P(k/k-1) - P(k/k) \quad (10)$$

Here, M is the width of the moving window.

Substituting the expression for the prediction error covariance matrix $P(k/k-1)$ into formula (10), we have

$$\begin{aligned} &\frac{1}{M}\sum_{j=k-M+1}^{k} \varepsilon(j)\varepsilon^T(j) = AP(k-1/k-1)A^T + \\ &BD_u(k-1)B^T + Q(k-1) - P(k/k) \end{aligned} \quad (11)$$

The system noise covariance matrix $Q(k-1)$ can be determined from expression (11) as:

$$\begin{aligned} &Q(k-1) = \frac{1}{M}\sum_{j=k-M+1}^{k} \varepsilon(j)\varepsilon^T(j) + P(k/k) \\ &-AP(k-1/k-1)A^T - BD_u(k-1)B^T \end{aligned} \quad (12)$$

In the case of $k \geq \tau$, in the sample state correction sequence covariance, a biased values $\varepsilon_b(k) = \varepsilon(k) + \Delta\varepsilon(k)$ are used instead of the unbiased value $\varepsilon(k)$, where $\Delta\varepsilon(k)$ is the state correction bias

$$\hat{S}_{\varepsilon b}(k) = \frac{1}{M}\sum_{j=k-M+1}^{k} \varepsilon_b(j)\varepsilon_b^T(j) \quad (13)$$

Remark. Note that the mean value of the state correction sequence $\varepsilon_b(k)$ in this case is not zero, so formula (13) is not a sample covariance. In the "sliding window," this is the mean square of state correction sequence (MSSCS). Bias type process noise change may be converted to the mean square of state correction sequence and such type of changes can be compensated using the covariance matching techniques.

Statement. The bias in state correction sequence for the $k \geq \tau$ iteration steps lead to an increase in the mathematical expectation of the mean square of state correction sequence.

Consequently, the process noise bias will increase the mathematical expectation of the mean square of state correction sequence. It can be seen from the *Theorem* and the *Statement* above that the process noise bias is transferred to the state correction sequence bias and changes the mean square of state correction sequence (13). The state correction sequence bias $\Delta\varepsilon(k)$ leads to a change in the values of $\varepsilon_b(k)$ and the elements of the mean square of state correction sequence. As a result, the bias in the process noise is transferred to the MSSCS. Thus, the expression (13) can be chosen as a statistic in the problem of compensating for changes in process noise.

Thus, the system noise bias will increase MSSCS (13). As a result, according to formula (12), the system noise covariance matrix $Q(k-1)$ will increase, resulting in a larger Kalman gain, which will increase the influence of measurements on the state update process and reduce the influence of the mathematical model of the system, consequently, reducing the influence of system noise bias. As a result, the robustness of the filter against the system noise bias is ensured and the deterioration of the estimation procedure caused by the system noise bias is prevented. However, the estimator for adapting the system noise covariance based on expression (12) is not recursive.

IV. Recursive System Noise Covariance Estimator

In order to predict the behaviour of the control, we need to find a recursion for $Q(k)$ based on $P(k+1/k)$. To predict the behavior of a control, we need to find a recursive version of the system noise covariance estimation based on the state correction sequence. Furthermore, updating the system noise covariance recursively is convenient and efficient since only the estimation value from the previous step k-1 and the latest filter parameters from step k are used in the recursive update process, which substantially simplifies the procedure and reduces the calculations. To construct such a recursion, it is enough to consider two successive expressions for the system noise covariance.

Expressions for the system noise covariance matrix for $k+1$ and k iterations respectively:

$$Q(k) = \frac{1}{M}\sum_{j=k-M+2}^{k+1} \varepsilon(j)\varepsilon^T(j) + P(k+1/k+1) - AP(k/k)A^T - BD_u(k)B^T \tag{14}$$

$$Q(k-1) = \frac{1}{M}\sum_{j=k-M+1}^{k} \varepsilon(j)\varepsilon^T(j) + P(k/k) - AP(k-1/k-1)A^T - BD_u(k-1)B^T \tag{15}$$

Therefore, the difference between the system noise covariances $Q(k)$ and $Q(k-1)$ can be written in the following form:

$$\begin{aligned} Q(k) - Q(k-1) &= \frac{1}{M}\times \\ &\left[\sum_{j=k-M+2}^{k+1} \varepsilon(j)\varepsilon^T(j) - \sum_{j=k-M+1}^{k} \varepsilon(j)\varepsilon^T(j)\right] + \\ &P(k+1/k+1) - P(k/k) - AP(k/k)A^T + \\ &AP(k-1/k-1)A^T - BD_u(k)B^T + BD_u(k-1)B^T \end{aligned} \tag{16}$$

After the necessary mathematical transformations, equation (16) can be rewritten as,

$$\begin{aligned} Q(k) &= Q(k-1) + \frac{1}{M}\times \\ &\left[\varepsilon(k+1)\varepsilon^T(k+1) - \varepsilon(k-M+1)\varepsilon^T(k-M+1)\right] - \\ &\left[P(k/k) - P(k+1/k+1)\right] + A\left[P(k-1/k-1) - P(k/k)\right] \\ &\times A^T + B\left[D_u(k-1) - D_u(k)\right]B^T \end{aligned} \tag{17}$$

The expression (17) can be written in a simple form through equations below:

$$Q(k) = Q(k-1) + \Delta Q(k) \tag{18}$$

$$\begin{aligned} \Delta Q(k) &= \frac{1}{M}\times \\ &\left[\varepsilon(k+1)\varepsilon^T(k+1) - \varepsilon(k-M+1)\varepsilon^T(k-M+1)\right] - \\ &\left[P(k/k) - P(k+1/k+1)\right] + A\left[P(k-1/k-1) - P(k/k)\right] \\ &\times A^T + B\left[D_u(k-1) - D_u(k)\right]B^T \end{aligned} \tag{19}$$

Equations (18) and (19) allow $Q(k)$ to be calculated recursively at each iteration. In the presence of system noise bias, according to formulas (12), (15), (18) and (19), the system noise covariance matrix $Q(k)$ will increase, resulting in a larger Kalman gain, which will increase the measurement impact of the state update process. Consequently, the influence of the mathematical model of the system, and hence the system noise, on the estimation of the Kalman filter is reduced. As a result, the robustness of the filter against the system noise bias is ensured and the deterioration of the estimation procedure caused by the system noise bias is prevented.

V. Simulation Results

The proposed covariance difference-based adaptive KF method is used to estimate the dynamics of an unmanned aerial vehicle platform. [13] presents a mathematical model for the UAV's combined longitudinal and lateral dynamics. For the estimation of the UAV state vector the AKF with recursive Q-adaptation and conventional KF are used. Simulations are realized in 1000 steps for a period of 100 seconds with 0.1 seconds of sampling time, Δt. System noise bias is simulated by adding the constant bias term $\lambda = \begin{bmatrix} 3 & 2 & 0.3 & 0.5 & 5 & 0.5 & 0.3 & 0.3 & 0.5 \end{bmatrix}^T$ to the system noise after 30th second. In the simulations, two longitudinal and two lateral parameters of the UAV motion are examined using two types of Kalman filters: AKF with recursive Q-adaptation and conventional KF. AKF with recursive Q-adaptation estimation results for the forward velocity, pitch rate, yaw rate and sideslip angle in the presence of *s*ystem noise bias are presented in Figs. 1-4. The graphs in these Figures show that the presented AKF with recursive Q-adaptation provides good results for estimating the UAV states in the presence of bias in the system noise.

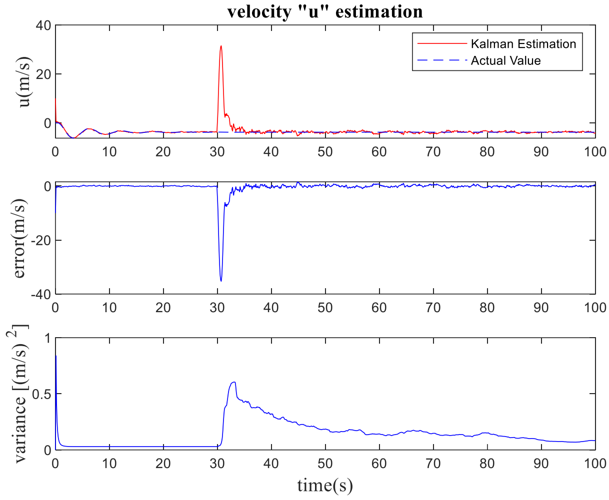

Fig. 1. Forward velocity estimation results using AKF with recursive Q-adaptation in the presence of *s*ystem noise bias

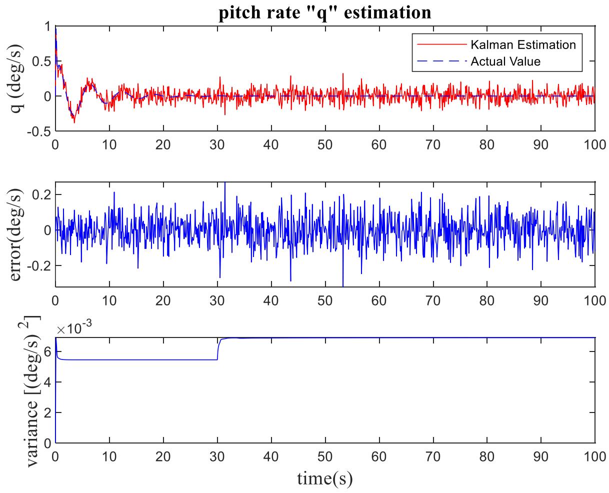

Fig. 2. Pitch rate estimation results using AKF with recursive Q-adaptation in the presence of *s*ystem noise bias

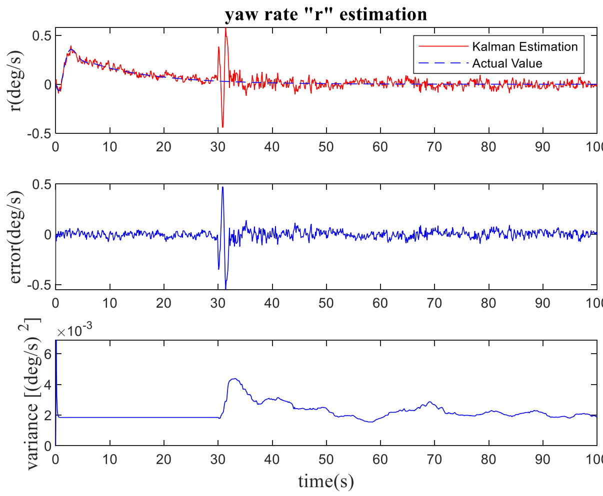

Fig. 3. Yaw rate estimation results using AKF with recursive Q-adaptation in the presence of *s*ystem noise bias

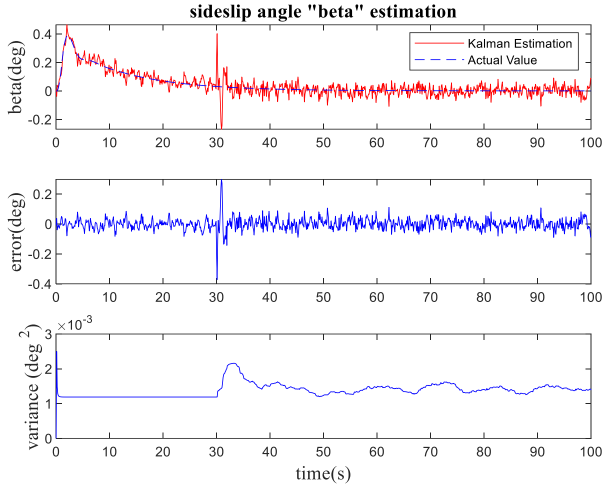

Fig. 4. Sideslip angle estimation results using AKF with recursive Q-adaptation in the presence of *s*ystem noise bias

The conventional KF estimation results for the pitch rate, pitch angle, roll rate and roll angle in the presence of system noise bias, are presented in Figs. 5 – 8 respectively.

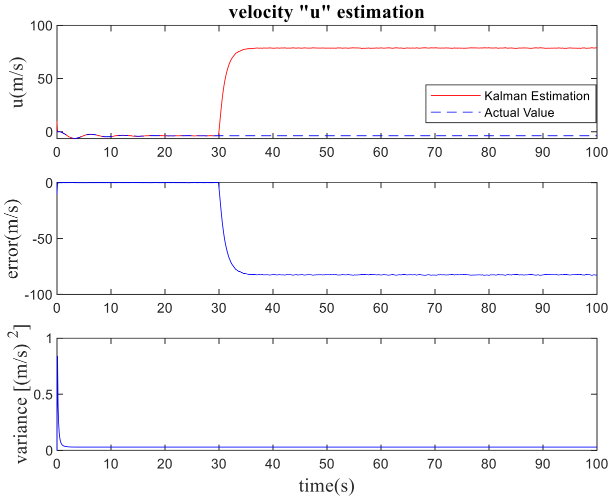

Fig. 5. Forward velocity estimation results using conventional KF in the presence of *s*ystem noise bias

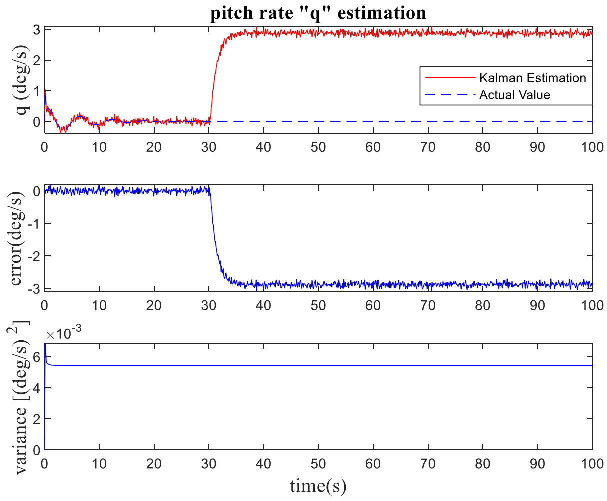

Fig. 6. Pitch rate estimation results using conventional KF in the presence of *s*ystem noise bias

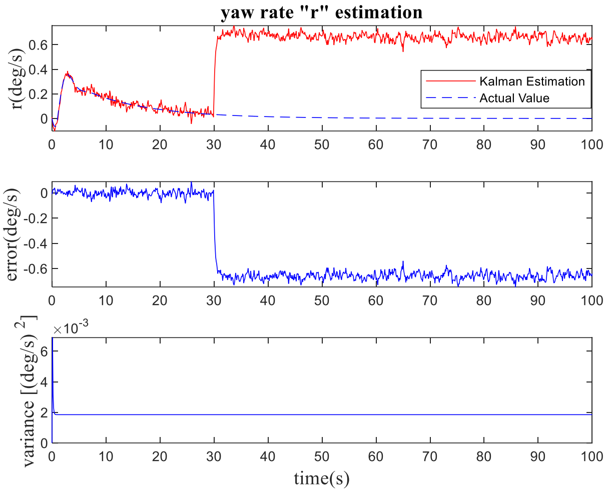

Fig. 7. Yaw rate estimation results using conventional KF in the presence of system noise bias

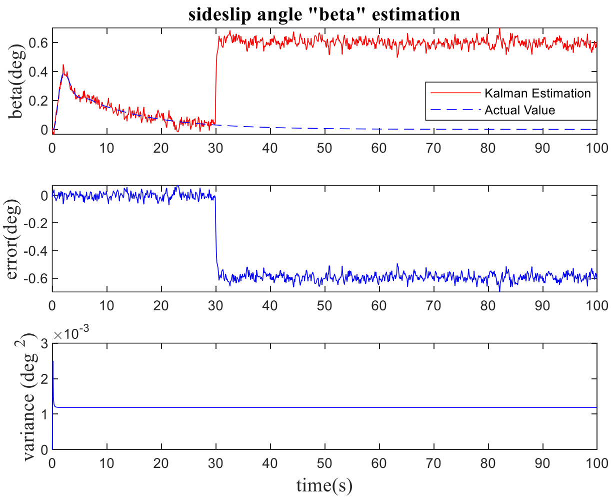

Fig. 8. Sideslip angle estimation results using conventional KF in the presence of system noise bias

As can be seen from the graphs presented in Fig. 5-8, conventional KF estimates diverge after the introduction of bias in the system noise. The conventional KF estimation results in the presence of system noise bias are unacceptable as the estimates of the UAV states.

The root mean square errors (RMSE) for the AKF with recursive Q-adaptation and conventional KF estimates calculated for the interval $400 \le k \le 1000$ in the presence of system noise bias are given in Table 1.

TABLE I. RMSE OF THE KALMAN FILTERS ESTIMATES IN CASE OF SYSTEM NOISE BIAS

Method	u **(m/s)**	q **(deg/s)**	r **(deg/s)**	β **(deg)**
AKF	0.5453, m/s	0.0848	0.0791	0.0330
CKF	82.5503	2.8795	0.6590	0.5955

As can be seen from the results presented in Table I, AKF with recursive Q-adaptation provides good estimation results compared to conventional KF in the presence of system noise bias. The RMS errors of AKF are significantly smaller than those of CKF. The estimation accuracy of the conventional KF is low.

VI. CONCLUSION

In this study a covariance difference-based adaptive Kalman filter with recursive Q-adaptation is presented. It is shown that the system noise bias type system changes will cause a change in the statistical characteristics of the state correction sequence of Kalman filter. For the purpose of estimating the UAV states, simulations are compared using the adaptive and non-adaptive versions of the Kalman filters in the presence of system noise bias type system changes.

In simulations, two types of Kalman filter algorithms were applied to the UAV dynamics model: the proposed AKF with recursive Q-adaptation and conventional KF. Simulation results show that the presented AKF with recursive Q-adaptation can accurately estimate UAV states in real time in the presence of system noise uncertainties and generally produces superior estimation results than conventional KF.

The AKF with recursive Q-adaptation is recommended as the reliable UAV state estimator in case of system noise uncertainties. The proposed method can be recommended for the flight control system of an UAV in point of view of supplying real-time system noise bias compensation.

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