Closed Transport Problem with Fuzzy Numbers

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Abstract: The paper examines the closed transport problem. It is assumed that the transportation of uniform goods from warehouses to destinations is not represented by fixed numbers, but by numbers within certain intervals. The endpoints of these intervals are provided accordingly. Based on these intervals, and in line with the economic essence of the problem, optimistic and pessimistic strategies are chosen. After that, the corresponding optimistic and pessimistic closed transport problems are formulated. By solving the resulting problems using any known method, the corresponding optimistic and pessimistic solutions of the original problem are obtained. It is clear that any solution of the transport problem with transportation costs in the form of intervals will lie between the optimistic and pessimistic solutions. Also, the minimal transportation cost of the interval transport problem will fall between the optimistic and pessimistic transportation costs. **(***Abstract***)**

Keywords: transport problem, transportation cost in the form of intervals, optimistic and pessimistic problems

I. Introduction

The economic essence of the known closed transport problem is as follows: Let's assume that there are *m* warehouses with corresponding quantities of uniform goods $a_i, (i = \overline{1,m})$ These goods need to be transported to *n* destinations (facilities). Additionally, assume that the transportation cost for moving one unit of goods from the *i*-th warehouse to the *j*-th destination is $c_{ij}, (i = \overline{1,m}\,, j = \overline{1,n})$. The problem is naturally formulated as follows.

The goods in the warehouses must be transported to the destinations in such a way that no goods remain in the warehouses, the demand of each destination is fulfilled as specified, and at the same time, the total transportation cost is minimized. Therefore, to formulate the mathematical model of the problem, let $x_{ij}, (i = \overline{1,m}\,, j = \overline{1,n})$ denote the unknown quantity of goods to be transported from the *i*-th warehouse to the *j*-th destination. Then, the total transportation cost for moving the goods from the warehouses to the destinations is:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

It is clear that this cost must be minimized.

According to the problem's constraints, the transportation should be carried out in such a way that no goods remain in the warehouses. For this, the system of equations

$$\sum_{j=1}^{n} x_{ij} = a_i, (i = \overline{1,m})$$

must be satisfied.

Another constraint is that the demand of each destination must be fully satisfied. This condition is ensured by the system of equations

$$\sum_{i=1}^{m} x_{ij} = b_j, (j = \overline{1,n}).$$

Thus, the following known mathematical model is obtained.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to min\,, \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, (i = \overline{1,m}) \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, (j = \overline{1,n}) \qquad (3)$$

$$x_{ij} \geq 0\,,\ \left(i = \overline{1,m}\,, j = \overline{1,n}\right) \qquad (4)$$

Note that in the (1 – 4) problem, we assume that the total quantity of goods in the warehouses is equal to the total demand of the destinations. That is

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

Therefore, this problem is called a closed transport problem [1].

The (1 – 4) problem has long been known, and certain methods have been developed to find its approximate and optimal solutions [1 - 3].

It should be noted that in the (1 – 4) problem, the transportation costs $c_{ij}, \left(i = \overline{1,m}\,, j = \overline{1,n}\right)$, the quantities $a_i, (i = \overline{1,m})$ in the warehouses, and the demands $b_j, (j = \overline{1,n})$ of the destinations have been treated as fixed and non-negative numbers as provided in the literature up to this point. However, in real-world practical problems, if we consider these given data as fixed numbers in the form of (1 – 4), the model might not adequately reflect the actual situation. This is because the transportation costs $c_{ij}, \left(i = \overline{1,m}\,, j = \overline{1,n}\right)$ could be adjusted slightly based on negotiations in real problems. Therefore, if we accept $c_{ij} \in \left[\underline{c_{ij}}\,, \overline{c_{ij}}\right]$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$, we obtain a more adequate model. Here, $\underline{c_{ij}}\,, \overline{c_{ij}}$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$ are the given fixed numbers. Note that the quantities $a_i, (i = \overline{1,m})$ and the demands $b_j, (j = \overline{1,n})$ should not be treated as numbers within intervals because that could lead to either excess goods remaining in the warehouses or an additional demand for goods. Therefore, we treat $a_i, (i = \overline{1,m})$ $b_j, (j = \overline{1,n})$as fixed numbers.

Thus, instead of the (1 – 4) problem, we obtain the following more general and adequate model of the transportation process.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \left[\underline{c_{ij}}\,, \overline{c_{ij}}\right] x_{ij} \to min\,, \qquad (5)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, (i = \overline{1,m}) \qquad (6)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \left(j = \overline{1,n}\right) \qquad (7)$$

$$x_{ij} \geq 0\,, \left(i = \overline{1,m}\,, j = \overline{1,n}\right) \qquad (8)$$

Here, $\underline{c_{ij}}\,, \overline{c_{ij}}, a_i\ and\ b_j$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$are given positive fixed numbers. These numbers can be treated as natural numbers without affecting the generality.

It should be noted that to solve the (5 – 8) problem, we need to solve transportation problems of the (1 – 4) type for each specified number in the interval $\left[\underline{c_{ij}}\,, \overline{c_{ij}}\right]$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$. In this case, the solution corresponding to the smallest value of the (1) function will be the optimal solution.

Clearly, since there are many numbers in the intervals $\left[\underline{c_{ij}}\,, \overline{c_{ij}}\right]$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$, we would need to solve a large number of (1 – 4) type problems. This approach is not practical and would require considerable time. Therefore, we have approached the solution of the (5 – 8) problem differently. Specifically, using [4 - 6], we have formulated optimistic and pessimistic problems based on the strategies.

In the optimistic problem, we accept $c_{ij} = \underline{c_{ij}}$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$, and in the pessimistic problem, we $c_{ij} = \overline{c_{ij}}$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$. The essence of the optimistic problem is to find the minimal transportation cost that satisfies the conditions (6 – 8) with the minimal $\underline{c_{ij}}$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$. In the pessimistic problem, we look for the variant with minimal total cost within the conditions (6 – 8) for the maximal $\overline{c_{ij}}$ $\left(i = \overline{1,m}\,, j = \overline{1,n}\right)$.

Thus, the following two problems must be solved.

I. Optimistic problem:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\underline{c_{ij}}x_{ij} \to min\,, \qquad (9)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, \left(i = \overline{1,m}\right) \qquad (10)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, (j = \overline{1,n}) \qquad (11)$$

$$x_{ij} \geq 0\,, \left(i = \overline{1,m}\,, j = \overline{1,n}\right) \qquad (12)$$

II. **Pessimistic problem**:

$$\sum_{i=1}^{m}\sum_{j=1}^{n}\overline{c_{ij}}\, x_{ij} \to min\,, \qquad (13)$$

$$\sum_{j=1}^{n} x_{ij} = a_i, (i = \overline{1,m}) \qquad (14)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, (j = \overline{1,n}) \qquad (15)$$

$$x_{ij} \geq 0\,, \left(i = \overline{1,m}\,, j = \overline{1,n}\right) \qquad (16)$$

As seen, both the optimistic problem (9 – 12) and the pessimistic problem (13 – 16) are known closed transport problems. Let's assume these problems are solved using any known method (e.g., the potential method), yielding the corresponding optimistic and pessimistic solutions

$$X^{op} = (x_{11}^{op}, x_{12}^{op}, \ldots, x_{mn}^{op})$$

$$X^{pes} = (x_{11}^{pes}, x_{12}^{pes}, \ldots, x_{mn}^{pes})$$

Then, we can calculate the transportation costs

$$f^{op} = \sum_{i=1}^{m}\sum_{j=1}^{n}\underline{c_{ij}}x_{ij}^{op}$$

and

$$f^{pes} = \sum_{i=1}^{m}\sum_{j=1}^{n}\overline{c_{ij}}x_{ij}^{pes}$$

If the optimal solution of the (5 – 8) problem is $X^* = (x_{11}^*, x_{12}^*, \ldots, x_{mn}^*)$ and the value of the (5) function for this solution is f^*, then it is clear that

$$f^{op} \leq f^* \leq f^{pes} \qquad (17)$$

It should be noted that finding f^* is not feasible, but f^{op} and f^{pes} can be easily found. Thus, the importance of the inequality (17) becomes evident.

Furthermore, any approximate solution to the optimistic (9 – 12) and pessimistic (13 – 16) problems, as well as the corresponding functional values, can easily be found using known methods (e.g., the minimal element method).

Conclusion: A more adequate mathematical model of the well-known transport problem, frequently encountered in practice, has been developed. In this model, the transportation costs are assumed to be within certain intervals, reflecting the reality more accurately. Based on this, closed optimistic and pessimistic problems were formulated. The resulting problems are easily solvable.

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