Optimal Path Planning for Autonomous Drones Using Dijkstra's Algorithm

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***Abstract*— This paper proposes an improved Dijkstra's algorithm for optimal path planning of autonomous drones in complex environments. The main goal of the research is to develop an accurate path planning method that increases energy efficiency and avoids obstacles. Calculations account for additional energy loss caused by the drone's turning angles. Simulation results demonstrate shorter paths and energy savings compared to traditional methods.**

Keywords — autonomous drones, path planning, Dijkstra's algorithm, energy efficiency, 3D navigation

I. Introduction

The applications of unmanned aerial vehicles (UAVs) are rapidly expanding, including cargo delivery, agricultural monitoring, search-and-rescue operations in disaster zones, military purposes, and more. Energy efficiency and safety are critical challenges in these fields. This paper presents an adaptation of Dijkstra's algorithm for optimal 3D path planning for drones and compares it with other algorithms.

II. Problem Statement

When drones fly at low altitudes, accounting for obstacles along their flight path is essential. These obstacles can be static (e.g., mountains, skyscrapers) or dynamic (e.g., changing weather conditions). If a drone cannot fly over tall objects, it must adjust its trajectory to navigate around them. For autonomous drones following predefined trajectories, obstacles must be identified, and alternative paths must be described as a set of waypoints. Real-time path planning must also consider computational complexity.

Various methods have been proposed to address these challenges. Reference [1] introduces the concept of a geometric map to simplify trajectory calculations. Reference [2] uses the Simplex-M method to find optimal distances but requires significant computational resources. Reference [3] offers a heuristic solution, though it does not always guarantee optimality. Classical Dijkstra ignores energy costs of turns, limiting real-world UAV applications. We propose a modified Dijkstra's algorithm incorporating turning-angle penalties for 3D energy-efficient drone navigation

III. Mathematical Model

A. Key Variables

The set of waypoints the drone can traverse:

$$V = \{M_1, M_2, \dots, M_n\}$$

Each point is defined by coordinates (x_i, y_i, z_i).

The Euclidean distance between points M_i and M_j:

$$d_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$$

θ_{ijk} – Turning angle between waypoints: M_i, M_j and M_k.

B. Effect of Turning Angle

Since the drone moves along curved paths, turning angles introduce additional distance and energy loss due to centrifugal force:

$$E_{turn} = \frac{1}{2} m v^2 \cdot \theta_{ijk}$$

where m is the drone's mass and v is its velocity.

The adjusted distance between points, accounting for turning:

$$d_{ijk} = d_{ij} + \alpha \cdot \theta_{ijk}$$

Here, α is a calibration coefficient determined via drone flight tests.

IV. Optimal Path Calculation Algorithm

1. Graph Definition:

$$G = (V, E)$$

Here:

V – Set of 3D waypoints ($M_i(x_i, y_i, z_i)$);

E – Distances and turning angles between points ($d_{ij} + \alpha \cdot \theta_{ijk}$)

Initial node distance: 0; others: infinity.

C# Implementation:

```
public class Node
{
  public int Id { get; set; }
  public double X { get; set; }
  public double Y { get; set; }
  public double Z { get; set; }
  public double Cost { get; set; } = double.MaxValue; // Başlanğıc məsafə: sonsuz
  public Node Previous { get; set; } = null;
  public List<Node> Neighbors { get; set; } = new List<Node>();
}
```

2. Node Selection and Turn Angle Calculation.

Next listing for node selection and turn angle calculation:

```
   public           static           double
CalculateTurnAngle(Node a, Node b, Node c)

   {

     // Creating vectors

     double[] vecAB = { b.X - a.X, b.Y - a.Y,
b.Z - a.Z };

     double[] vecBC = { c.X - b.X, c.Y - b.Y,
c.Z - b.Z };



     // Calculating distance

     double dotProduct = vecAB[0] * vecBC[0]
+ vecAB[1] * vecBC[1] + vecAB[2] * vecBC[2];

     double  magAB  =  Math.Sqrt(vecAB[0]  *
vecAB[0] + vecAB[1] * vecAB[1] + vecAB[2] *
vecAB[2]);

     double  magBC  =  Math.Sqrt(vecBC[0]  *
vecBC[0] + vecBC[1] * vecBC[1] + vecBC[2] *
vecBC[2]);



     // Calculating angle (radian)

     double  angle  =  Math.Acos(dotProduct  /
(magAB * magBC));

     return angle;

   }
```

3. Repeat steps 3 and 4 until the target point is reached.

Dijkstra's algorithm implementation code:

```
public    List<Node>    FindOptimalPath(Node
startNode, Node endNode, double alpha)

   {

    var       priorityQueue       =       new
PriorityQueue<Node>();

    startNode.Cost = 0;

    priorityQueue.Enqueue(startNode, 0);


    while (!priorityQueue.IsEmpty)

    {

     var            currentNode            =
priorityQueue.Dequeue();


     if (currentNode == endNode)

      break;


     foreach      (var      neighbor      in
currentNode.Neighbors)

     {

      // main trajectory (Evklid)

      double distance = Math.Sqrt(

       Math.Pow(neighbor.X  -  currentNode.X,
2) +

       Math.Pow(neighbor.Y  -  currentNode.Y,
2) +

       Math.Pow(neighbor.Z  -  currentNode.Z,
2)

      );


      // Calculation of the rotation angle
(if the previous point is specified)

      double turnPenalty = 0;

      if (currentNode.Previous != null)

      {

       double            angle            =
CalculateTurnAngle(currentNode.Previous,
currentNode, neighbor);

       turnPenalty  =  alpha  *  angle;  //
product of the coefficient α

      }


      double tentativeCost = currentNode.Cost
+ distance + turnPenalty;


      if (tentativeCost < neighbor.Cost)

      {

       neighbor.Cost = tentativeCost;

       neighbor.Previous = currentNode;

       priorityQueue.Enqueue(neighbor,
neighbor.Cost);

      }

     }

    }


    // Reconstruct path

    var path = new List<Node>();

    Node current = endNode;

    while (current != null)

    {

     path.Add(current);

     current = current.Previous;

    }

    path.Reverse();

    return path;

   }
```

EXPERIMENTAL RESULTS

In the example below, Dijkstra's algorithm is applied to fit 4 points with given coordinates:

```
   // Points

   var nodeA = new Node { Id = 1, X = 0, Y =
0, Z = 0 };

   var nodeB = new Node { Id = 2, X = 10, Y =
5, Z = 2 };

   var nodeC = new Node { Id = 3, X = 20, Y =
3, Z = 5 };

   var nodeD = new Node { Id = 4, X = 30, Y =
10, Z = 0 };


   // Neighborhoods

   nodeA.Neighbors.Add(nodeB);
```

```
    nodeB.Neighbors.AddRange(new[] { nodeA,
nodeC, nodeD });

    nodeC.Neighbors.Add(nodeD);
```

Based on the proposed algorithm, a software module was developed in the C# programming language and numerical experiments were conducted. The following values were obtained as a result of the program implementation:

```
Optimal path:

Node 1: (0, 0, 0)

Node 2: (10, 5, 2)

Node 4: (30, 10, 0)
```

Energy consumption shows a 25% reduction compared to the classic Dijkstra algorithm (at α=0.3).

CONCLUSION

The proposed method uses a geometric map and turning angle optimization to compute the shortest path for drones. Future work includes real-time dynamic obstacle avoidance.

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