Mathematical Models for Determining the Temperature on the Surface of Mineral Waters

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***Abstract* - Azerbaijan situated in the east of the Caucasian region of the Alpine folding beltisrich in not only great oil, gas and mineral reserves, but in unique mineral water deposits as well. Nonetheless, exploration of new potable, mineral and thermal water sources persists as one of the vital targets of geological services of the Republic. The issue of efficient use of available water resource assumes, at the same time, more and more significant attitude. This article is dedicated to ways solve this problem. In addition, we consider the inverse problem of determining the temperature of geothermal waters as sources of temperature dependence on the surface. We are talking about those tasks in which it is necessary to determine the causes if the results obtained because of observation are known. For example, to determine the location and power of an earthquake from the oscillations measured on the surface of the earth. Or by the structure on the surface of the earth's minerals to determine the depth of their location underground, etc. When processing these experiments, a conclusion is drawn about the internal relationships of a phenomenon or process. When the mathematical model of the process under study is known, the problem of identifying the mathematical model is posed. In general, inverse problems are understood as tasks in which it is possible to draw conclusions in determining the parameters of this model from the available results of observations or other experimental information. The study is devoted to the inverse problem of determining the temperature of geothermal waters depending on the surface temperature for the efficient use of available water resources.**

Keywords - geothermal, debit, mineral content, chemical, gradient, inverse problem, differential equations, mathematical model

I. INTRODUCTION

When studying physical objects or phenomena by experimental methods, a typical situation is when the quantitative characteristics of the object of interest to the researcher are not available for direct observation. Or carrying out the experiment itself is generally impossible, because it is either prohibited or too dangerous. Also, an experiment can be associated with very high financial costs. But, it is practically possible to obtain some indirect information about the object under study and from which to draw a conclusion about the properties of the object or process being studied. This information is determined by the nature of the studied object and the experimental complex used in this study. In such situations, for the diagnosis of objects and their internal structure, mathematical processing and interpretation of the observation results is required.

We are talking about those tasks in which it is necessary to determine the causes if the results obtained as a result of observation are known. For example, to determine the location and power of an earthquake from the oscillations measured on the surface of the earth. Or by the structure on the surface of the earth's minerals to determine the depth of their location underground, etc. When processing these experiments, a conclusion is drawn about the internal relationships of a phenomenon or process. When the mathematical model of the process under study is known, the problem of identifying the mathematical model is posed. Questions in this direction can be found in [1]-[9]. In general, inverse problems are understood as tasks in which it is possible to draw conclusions in determining the parameters of this model from the available results of observations or other experimental in formation.

Namely, the determination of the coefficients of differential equations, their right- hand side, boundary of the domain, boundary and initial conditions. These problems relate to inverse problems of mathematical physics. Individual mineral associations occurring in the form of various types of rocks are a natural process and are the product of a physical and chemical system formed in certain geological conditions.

Azerbaijan is a country with its picturesque landscape, curative climate with its mud volcanoes, high mountains, numerous lowlands, unique Naphtalan oil and golden seaside beaches and countless mineral and thermal water sources.

The resort business in our Republic has recently become one of major branches of national public health services, for implementation of improvement measures, oriented on improvement of strengthening of health of the population, sporting and development of tourism.

In ancient times, at the dawn of medicine, Azerbaijan had been using thermal and mineral natural sources for treatment of illnesses -in Istisu, Turshsu, Naphthalan, Surakhani, Asrikderesy, Ibadisu, Meshasu, Gotursu, Chukhouryurd, Elisu and in other ancient bath-houses and primitive tubs, which were directly set up in places of an output of warm and hot mineral waters [10] .

Numerous clinical experimental researches, established high balneal efficiency of mineral waters of the Republic (Istisu, Sirab, Turshsu, Surakhani, Shikova, Meshasu, Daridag, Arkevan, Galaalti and many other). Mineral waters with a daily production rate of over 100 *mln*. Liter sensually erupt around 300ths tons of different salts on a day surface of the Earth that can be widely utilized in chemical industry, pharmacology and different branches of economics.

Now scientists from many countries are anxious about that the non-renewable fuel resources of our planet steadily exhaust, and in connection with rapid development of industry and agriculture in XXI century consumption of fuel energy will grow extremely with fast paces. Therefore further fundamental and applied scientific researches on usage of alternative energy sources -solar, wind and thermal waters - gain today the relevant and prime value.

The Azerbaijan Republic has considerable reserves of thermal waters. Thermal energy of thermal waters, including open ten thousand earlier drilled oil and gas well, can be successfully utilized in different industries and agriculture. The underground thermal waters are main storage and carrier means of plutonic heat, due to their mobility and greatest thermal capacity.

II. INVERSE PROBLEM

Consider the following mathematical model of the process under study in the form of an initial - boundary value problem for the heat equation

$$c \cdot u_t = (ku_x)_x - qu + f(x) \tag{1}$$

$(x,t) \in Q_T = \Omega \times (0,\mathbf{T}), \Omega$ **– some bounded,** $\Omega \subset R^n, n \geq 2$, with a smooth boundary, and the variable $t \in (0,T)$. The coefficients included in the equation, as well as

$$u(x,0 = \varphi(x) \quad x \in \Omega \tag{2}$$

$$u(x,t = \psi(x,t) \quad (x,t) \in S_T \tag{3}$$

where, $S_T \in \partial\Omega \times (0,\mathbf{T})$ and the initial condition (2) and the boundary condition (3) are some effective characteristics of the process under study. We will study the temperature of geothermal waters at the source, thanks to the temperature at the surface.

In the case when the task describes the process distribution of the flow of volcanic heat in a certain direction, the coefficients c and k are respectively the heat capacity and thermal conductivity coefficients and characterize process geothermal water movements the composition of underground rocks. The function *f* is the density of heat sources. All other functions included in problem [11], [12], [13] also have thermo physical interpretation. From an experiment using sensors or knowing the temperature of the melting of rocks on the surface at the point x_0, the function is determined

$$g(t) = u(x_0, t)$$

Thus, the inverse problem arises. Namely, to determine the thermal conductivity of rocks underground, if the function $g(t)$ is specified. Other cases can also be considered, depending on the task. Here various types of assignment of additional information play an important role, i.e. the nature of the experiments. For simplicity, we consider the equation

$$u(t) - \Delta u = f(x)h(x,t) + g(x,t), \tag{4}$$

here is the $\mathbf{\Delta}$ - Laplace operator, i.e. $\Delta = \sum_{i=1}^{n} \partial^2 / \partial x_i^2$

We assume that the functions *h (x, t), g (x, t)* are

known. Unknown functions *f (x)* and *u (x, t)*. These functions must be defined using an additional condition, for example, a condition

$$\int_0^T u(x,\tau)d\tau = \varphi_1(x) \quad x \in \Omega \tag{5}$$

where the function $\varphi_1(x)$ is known. Thus, we obtain the following inverse problem

$$u_t - \Delta u = f(x)h(x,t) + g(x,t) \quad (x,t) \in Q_T \tag{6}$$

$$u(x,0) = \varphi(x) \quad x \in \Omega \tag{7}$$

$$u(x,t) = \psi(x,t) \quad (x,t) \in S_T \tag{8}$$

where functions *h(x, t), g(x, t),* $\varphi(x)$, $\psi(x,t)$, $\varphi_1(x)$ - are given functions and depend on the nature of the problem. Unknown functions are functions *u(x, t)* and *f (x)*. The solutions of this problem is exists and unique.

III. MATERIALS AND METHODS

In connection with continuous growth of the world power consumption and gradual exhaustion of its conventional sources, such as oil, gas, black coal, attention of scientists is centered upon searching new energy sources. We consider that in Azerbaijan's circumstances thermal waters, alongside with wind and solar energy are valuable as well the advantage of thermal waters is, that their reserves continuously renew, there is a capability to obtain heat, energy directly in place. They are valuable for curative properties and capabilities of obtaining valuable chemical elements.

These days, in connection with crisis of fuel and energy resources, wider use of the Earth's plutonic heat for electric power production (Italy, Iceland, New Zealand, USA, Japan, Bulgaria, Czechoslovakia, Hungary and other) in agriculture [11], municipal services, chemical industry, and for medical purposes as well has started abroad.

IV. DISCUSSION

Today to produce cost-effective electric power it is expedient to use temperature of a heat carrier not below 80^0C. Azerbaijan Republic is rich in thermal waters, which are known in a number of regions of the Greater and Lesser Caucasus, Absheron peninsula, Talish, and in the vast Kur lowland and Precaspian-Guba areas. A number of wells had been drilled for oil and gas and no hydrocarbons were discovered. These wells could be used for production of thermal waters within the above mentioned areas. Nowadays negotiations are being conducted with Azerbaijan Agency on Alternative and Renewable Energy for production of electrical power from high temperature well waters. The Lesser Caucasus introduces especial concern regarding a geothermal mode. Ancient thermal water sources have always been well known in its various his parts. These waters are mainly associated with Quaternary rocks of magmatic nature.

The known resort zone Istisu (Kalbajar region) is stretched more than 40 km along the Terter river is characterized by an abnormal thermal environment. The inverse geothermal gradient on southern slope (health resort Istisu and Bagirsakh field) is reduced up to 2-5 *m* and less and for the entire resort region is close to 18 *m*, i.e. much less than the average for the earth crust. The area is complicated by large tectonic faults and numerous carbon dioxide shows are observed there in. According to data obtained from numerous wells drilled in the area, temperature of thermal waters on Bagirsakh field is fast increasing and reaches 80˚C at depth of about 100*m*. The total production rate of water in region of the Upper Istisu is 800-900 ths*l/day*, Lower Istisu – 25 ths *l/day*. Elemental composition of water is of carbon, chloride, sulphide, hydrocarbonate, sodium structure. The thermal waters in Masalli, Lenkeran and Astara regions are characterized by a regional fault intersecting the entire mountainous Talish.

Waters with 44-65˚C temperature are encountered at depth of 500m in wells drilled in Arkevan water field in Masalli region. Temperature of waters in different sources of this region changes from 50˚C up to 64˚C. Production rate of wells is 10-15 *1/sec*. Water mineralized (17-18 g/l) chloride-calcium structure. In Lenkeran area (region of Meshasu, Ibadisu, Gavzavua and Khavtxoni sources) a number of wells with depth 465-1000*m* were drilled, which have opened waters with temperature up to 50^0C. Temperature of water in sources 30-43^0C, production rate up to 10*1/sec*. In regions of sources Astara wells with depth 300-500 *m* opened thermal waters with temperature 35^0C-50^0C. Water mineralization reaches 18-29 *g/l* and they are of sodium chloride content. The total production rate of sources in wells in Talish is 23625 m^3/day.

In Precaspian-Guba zone (southern slope of Greater Caucasus) 8 drilled wells opened thermal waters with a total production rate 112360 m^3/day, temperature 50^0C - 84^0C. In Khacmaz region a single thermal water well only has a production rate 1228 m^3/day, with temperature 58^0C. Thermal waters with temperature 50-81^0C, with a total production rate 30000 m^3/day have been obtained from prospecting boreholes in Precaspian-Guba zone from Mesocenozoic deposits. A single well 3 reached thermal waters with temperature 81^0C (on the surface) and with a production rate 4500 m^3/day. Temperature change as a function of depth in the area is reflected.

On Absheron peninsula thermal waters are encountered in wells at different depths. Thus, temperature of salt waters to the east of Hovsan village from the drilled wells reaches 100^0C.In Bibi-Heybat, which is immediately close to Baku city, waters with salinity 16,5 *g/l*, with temperature 71^0C and production rate 450 ths*1/day* are of chloride hydrocarbonate sodium content. There are large artesian basins with composite distribution of temperature (with high temperature manifestation) and structure of water in the Kur lowland [15], [16].

These thermal waters are associated with Absheron age deposits, have high pressure and are of hydrocarbonate sodium content. The Kur lowland has fair supplies of thermal waters, it is possible to use them completely in a cost-effective way and with the purposes of heating the civil and industrial facilities, obtain chemically rare elements and also in the balneal purposes.

Many wells drilled for oil and gas in Babazanan, Neftchala, Khilly, Mishovdag were void and stroke thermal waters, instead. There is a well 3 in Jarli field (Kurdemir region) with depth of 3050*m* and with production rate of 20000 m^3/day and temperature reaching 100˚C on the surface [17].

The thermal waters of Azerbaijan, as a whole, are not involved in industrial development. They will be used with primitive application for balneal purposes only. However, there is a positive experience, when in the end of the XX century 10 green houses with use of thermal waters were built in Lenkeran and crops are yielded 2 times a year in the above mentioned green houses.

2 pilot farms were established in the end of 60s of the last century in Astara-Lenkeran-Masalli area, each of which possessed green houses with metal pipes of 40-55*mm* in diameter, where hot water circulated and heated air and soil. Spacing interval between adjunct pipes was approximately 100 *sm*., 5 greenhouses with total area of 700m^2 were constructed in Masalli region. Water from drilled wells with temperature 44-60^0C and total production rate of 5-6 *1/sec*. Cucumber and tomato sprouts were planted in December 1967 and crop was obtained in March 1968.

In Alashin - region of thermal sources (Astara region) 4 greenhouses with total area of 1000 m^2 were constructed. Water, with temperature 45-58^0C and total production rate 15-8 *1/sec*. Sprouts of tomato and cucumber had been planted in February, 1968, the crop is obtained in the beginning of May.

The first experience of usage of thermal waters for heating of greenhouses has shown large prospects of the method and also the capability of obtaining 2-3 crops annually at minimum costs.

It is necessary to separately note that there are more 1000 natural outputs of mineral waters in Azerbaijan. More than 50% of these sources has temperature more than 42^0C, that demonstrates their considerable temperature at depth and mixing of thermal waters with cold waters in the upper horizons.

The usage of underground mineral waters in Azerbaijani resorts play an important role in recovery capacity for people suffering from diseases, such as, to be necessarily noted, atherosclerosis, idiopathic hypertension, coronary failure etc., and a number of valuable fields of mineral waters have temporarily remained in lands occupied by Armenia.

All the above mentioned give grounds for statement on necessity of acceptance of complex measures on involvement of huge resources of underground mineral and thermal waters of Azerbaijan in national economy.

Underground waters, Including, mineral and thermal waters, are in valuable in development of chemical industry. They can be utilized with the purposes of obtaining of a number of chemical agents (boric acid, iodine, bromine, strontium etc.).Some water deposits in Nakhchivan (Sirab, Daridag) already produce carbon dioxide. Taking into account high concentration of rare chemical elements (arsenic, lithium, antimony, selenium) their production is possible as well.

Thermal waters can be used in heating civil and industrial premises ($T= 40\text{-}60^0C$). It was put in topractice in Talish-Lenkeran region while heating green houses and administrative premises using waters with temperature 40-66^0C, production rate 160000 m^3/day from depth of 200-1000*m*.

These waters can be wide lyused for curing various diseases. They are currently being used in low volume in Kalbajar (Istisu), Masalli (Yeddi Gardash), Kurdemir (Jarli), Shamakhi (Chukhur-Yurd).

Extraction of *I, Br, B, Sr, Mg* and other chemical elements from thermal water siscommercially advantageous. Electrical energy can be produced from waters with temperatures above 80^0C (Jarli, Precaspian-Gubaetc.).

Presence of thermal waters distribution pattern in time and space associated with tectonic faults and magmatic processes has been proven from the scientific point of view.

These water sare the source of reliable information in elaboration of the history of geologic development of theare a, what cannot be denied.

V. CONCLUSION

The exploration of new potable, mineral and thermal water sources persists as one of the vital targets of geological services of the Republic. The issue of efficient use of available water resource assumes, at the same time, more and more significant attitude. This article is dedicated to ways solve this problem. In addition, we consider the inverse problem of determining the temperature of geothermal waters as sources of temperature dependence on the surface. We are talking about those tasks in which it is necessary to determine the causes if the results obtained because of observation are known. For example, to determine the location and power of an earthquake from the oscillations measured on the surface of the earth. Or by the structure on the surface of the earth's minerals to determine the depth of their location underground, etc. When processing these experiments, a conclusion is drawn about the internal relationships of a phenomenon or process. When the mathematical model of the process under study is known, the problem of identifying the mathematical model is posed.

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