Solution of the Synthesis Problem for Some Classes of Binary Multiparameter Nonlinear Modular Dynamic Systems

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Abstract: **A problem of synthesis the 3D – and 4D – binary nonlinear modular dynamical system with fixed memory, limited connection and a given degree, which descripiton in form of two valued analogue of Volterra's polynom is considered. The synthesis problem is reduced to a matrix-vector for. When the input sequences of the system satisfy the conditions orthogonality, an algorithm for solving the problem is given. If the input sequences do not satisfy the conditions orthogonality for solving the problem, a technique based on the use of special orthogonal input sequences is proposed**

Keywords: 3D – and 4D – binary nonlinear modular dynamic system; Volterra's polynomial; conditions of orthogonality*.*

INTIRODUKTION

Modular dynamic systems (MDS) are widely used in computer technology, diagnostic systems, encoding and decoding of discrete messages, cryptography, modeling, control of continuous and discrete objects, etc. In this application, conventional MDS and 2D-MDS are mainly used. However, there are some classes of MDS that have a more general structure than 2D-MDS. These classes include 3D-NMDC and 4D-NMDS. Note that when studying these systems, effective results can be obtained. Therefore, this work addresses the issue of solving the synthesis problem for 3D-NMDS and 4D-NMDS.

1. Solution of binary the synthesis problem for 3-Dimensional nonlinear modular dynamic system (3D-NMDS) with fixed memory n_0, limited communication $P = P_1 \times P_2$ and degree S are described in the form of two valued analogue of Volterra's polynomial is considered [1]:

The formula binary 3D-NMDS with fixed memory n_0, limited communication $P = P_1 \times P_2$ and degree S are following:

$$y[n,c_1,c_2] = \sum_{i=0}^{S} \sum_{\nu=1}^{\lambda_i} \sum_{\bar{j}\in L_1(\ell_1)} \sum_{\bar{\sigma}\in L_2(\ell_2)} \sum_{\bar{\bar{\tau}}\in\Gamma(i,\nu)} h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\bar{\tau}}]\times$$
$$\times \prod_{(\alpha,\beta,\gamma)\in Q_1(i,\nu)} \upsilon_{i,\nu}[n-\tau(\alpha,\beta,\gamma), c_1+p_1(j_\alpha), c_2+p_2(\sigma_\beta)], \qquad (1)$$

$$GF(2),$$

$$n\in[0,\mathrm{N}]\equiv\{0,1,\ldots\mathrm{N}\},\quad c_1\in[0,C_1]\equiv\{0,1,\ldots C_1\},$$
$$c_2\in[0,C_2]\equiv\{0,1,\ldots C_2\};$$
$$P_\alpha=\{p_\alpha(1),\ldots,p_\alpha(r_\alpha)\},-\infty<p_\alpha(1)<\ldots<p_\alpha(r_\alpha)<+\infty,$$
$$p_\alpha(\eta)\in\{\ldots,-1,0,1\ldots\},\quad \eta=1,\ldots,r_\alpha,\quad \alpha=\overline{1,2};$$
$$\bar{j}=(j_1,\ldots,j_{\ell_1}) \text{ and } \bar{\sigma}=(\sigma_1,\ldots,\sigma_{\ell_2})$$

are sets respectively sets.

$$L_1(\ell_1) \text{ and } L_2(\ell_2),$$

where

$$L_1(\ell_1)=\{(j_1,\ldots,j_{\ell_1})\,|\,1\le j_1<\ldots<j_{\ell_1}\le r_1\},$$
$$L_1(\ell_2)=\{(\sigma_1,\ldots,\sigma_{\ell_2})\,|\,1\le \sigma_1<\ldots<\sigma_{\ell_2}\le r_2\};$$

λ_i is the number of elements of the set

$F(i)$, and there $(\ell_1,\ell_2,\bar{m})_\nu$ is the ν - the element of sets $F(i)$, where there are the following sets

$$F(i)=$$
$$=\{(\ell_1,\ell_2,\bar{m}):\ \bar{m}=(m_{1,1},\ldots,m_{1,\ell_1},m_{2,1}\ldots,m_{2,\ell_2},\ldots,m_{\ell_1,\ell_2}),$$
$$\sum_{\alpha=1}^{\ell_1}\sum_{\beta=1}^{\ell_2} m_{\alpha,\beta}=i;\ \ m_{\alpha,\beta}\in\{0,\ldots,n_0+1\},\ \alpha=\overline{1,\ell_1},\beta=\overline{1,\ell_2}\};$$

$$(\forall\alpha\in\{1,\ldots,\ell_1\})(\exists\beta\in\{1,\ldots,\ell_2\})\Rightarrow(m_{\alpha,\beta}\ne 0),$$
$$(\forall\beta\in\{1,\ldots,\ell_2\})(\exists\alpha\in\{1,\ldots,\ell_1\})\Rightarrow(m_{\alpha,\beta}\ne 0),$$
$$\ell_\gamma\in\{1,\ldots,r_\alpha\},\gamma=\overline{1,2}\};$$
$$Q_1(i,\nu)=$$
$$=\{(\alpha,\beta,\gamma)\ :\gamma\in\{1,\ldots,m_{\alpha,\beta}\},(\alpha,\beta)\in Q_0(i,\ell_1,\ell_2,\bar{m})\},$$

where

$$Q_0(i,\ell_1,\ell_2,\bar{m})=\{(\alpha,\beta):\ m_{\alpha,\beta}\ \textit{iz kompotens}$$
$$\textit{of}\ \ \bar{m}\ \textit{and}\ \ m_{\alpha,\beta}\ne 0,\alpha=\overline{1,\ell_1},\beta=\overline{1,\ell_2}\},$$
$$Q(i,\bar{m})=\{(\alpha,\beta): m_{\alpha,\beta}\ \textit{is komponent of}\ \bar{m}\ \textit{and}$$
$$m_{\alpha,\beta}\ne 0\ \textit{is component of and}\ m_{\alpha,\beta}\ne 0,$$
$$\alpha=\overline{1,r_1},\beta=\overline{1,r_2}\};$$

$\Gamma(i,\nu)$ there are block vectors of the form $\bar{\bar{\tau}}$ and is formed from is $\bar{\tau}_{\alpha,\beta}$, where $(\alpha,\beta)\in Q(i,\overline{m})$, $i\in\{1,...,(n_0+1)r_1r_2\}$, $\overline{m}\in\Phi(i)$,

$$\Phi(i)=$$

$$=\{\overline{m}=(m_{1,1},...,m_{1,r_2},...,m_{r_1,r_2}):m_{\ell,k}\in\{0,...,n_0+1\},$$

$$\ell=\overline{1,r_1},\ k=\overline{1,r_2}\ ,\ m_{1,1},...,m_{1,r_2},...,m_{r_1,r_2}=i\},$$

$$\Gamma(i,\nu)=\underset{(\alpha,\beta)\in Q_1(i,\nu)}{\times}\Gamma_{\alpha,\beta}(m_{\alpha,\beta})\ ,$$

$$\Gamma_1(m_{\alpha,\beta})=\{\bar{\tau}_{\alpha,\beta}=(\tau(\alpha,\beta,1),...,\tau(\alpha,\beta,m_{\alpha,\beta})):\ 0\le$$

$$\le\tau(\alpha,\beta,1)<...<\tau(\alpha,\beta,m_{\alpha,\beta})\le n_0\};$$

Sequence

$$\{\upsilon_{i,\nu}[n,c_1,c_2]:n\in[0,N],c_1\in[0,C_1],c_2\in[0,C_2]\}\quad(2)$$

is an arbitrary binary sequence.

Sequence (2) is an arbitrary binary sequence. The task of synthesizing binary $3D$ - NMDS is to find such a value of impulse characteristics $h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\tau}]$ for each $\bar{j}\in L_1(\ell_1)$, $\bar{\sigma}\in L_2(\ell_2)$, $\nu=1,...,\lambda_i$, $i=0,...,S$, $3D$ - NMDS (1) at which the functional is minimized functional

$$J=\sum_{n=0}^{N}\sum_{c_1=0}^{C_1}\sum_{c_2=0}^{C_2}(y[n,c_1,c_2]-y_0[n,c_1,c_2])^2\qquad(3)$$

where

$$\{y_0[n,c_1,c_2]:n\in[0,N],c_1\in[0,C_1],c_2\in[0,C_2]\}$$

there is the desired output sequence. Problem (1), (3) has the following matrix-vector view:

$$Y=V\cdot H,\ GF(2),\qquad(4)$$

$$J=(Y-Y_0)^T\cdot(Y-Y_0)\to\min\qquad(5)$$

where

$$Y_0=(y_0[0,0,0],...,y_0[0,0,C_2],...,y_0[0,C_1,C_2],...$$

$$...,y_0[N,C_1,C_2])^T,$$

$$Y=$$

$$=(y[0,0,0],...,y[0,0,C_2],...,y[0,C_1,C_2],...,y[N,C_1,C_2])^T,$$

and the block matrix is formed from the input sequence (2) according to the following sequencea block matrix. V formed from the input sequence (2) bay following: sequence

$$V_0(i,\nu,\bar{j},\bar{\sigma},\bar{\bar{\tau}}_\xi)=$$

$$=\prod_{(\alpha,\beta,\gamma)\in Q_1(i,\nu)}\upsilon_{i,\nu}[n-\tau^{(\xi)}(\alpha,\beta,\gamma),c_1+p_1(j_\alpha),c_2+p_2(\sigma_\beta)],$$

$$V_1(i,\nu,\bar{j},\bar{\sigma})=$$

$$=(V_0(i,\nu,\bar{j},\bar{\sigma},\bar{\bar{\tau}}_1)\ldots V_0(i,\nu,\bar{j},\bar{\sigma},\bar{\bar{\tau}}_{|\Gamma(i,\nu)|}))\ ,$$

$$V_2(i,\nu,\bar{j})=(V_1(i,\nu,\bar{j},\bar{\sigma}_1)\ \ldots\ V_1(i,\nu,\bar{j},\bar{\sigma}_{|L_2(\ell_2)|}))\ ,$$

$$V_3(i,\nu)=(V_2(i,\nu,\bar{j}_1)\ \ldots\ V_2(i,\nu,\bar{j}_{|L_1(\ell_1)|}))\ ,\qquad(6)$$

$$V_4(i)=(V_3(i,1)\ \ldots\ V_3(i,\lambda_i))\ ,\ V=(V_4(1)\ \ldots\ V_4(s))\ ,$$

where $\bar{\bar{\tau}}_\xi$ is ξ - the element in set $\Gamma(i,\nu)$, components of vectors (recruitments) $\bar{\bar{\tau}}_\xi$ is $\tau^{(\xi)}(\alpha,\beta,\gamma)$, $\gamma\in\{1,...,m_{\alpha,\beta}\}$, $(\alpha,\beta)\in Q_0(i,\ell_1,\ell_2,\overline{m})$, and there $U_0(i,\nu,\bar{j},\bar{\sigma},\bar{\bar{\tau}}_\xi)$ is $(N+1)(C_1+1)(C_2+1)\times1$ - dimensional matrix. The block vector column is constructed bay following sequence:

$$H_1(i,\nu,\bar{j},\bar{\sigma})=(h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\bar{\tau}}_1],...,h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\bar{\tau}}_{|\Gamma(i,\nu)|}])^T$$

$$H_2(i,\nu,\bar{j})=(H_1(i,\nu,\bar{j},\bar{\sigma}_1),...,H_1(i,\nu,\bar{j},\bar{\sigma}_{|L_2(\ell_2)|}))^T\ ,$$

$$H_3(i,\nu)=(H_2(i,\nu,\bar{j}_1),...,H_2(i,\nu,\bar{j}_{|L_1(\ell_1)|}))^T\ ,\qquad(7)$$

$$H_4(i)=(H_3(i,1),...,H_3(i,\lambda_i))^T\ ,\quad H=(H_4(1),...,H_4(s))^T.$$

In (4) block matrix V and the block vectors are have size according is with $(N+1)(C_1+1)(C_2+1)\times r$ and $r\times1$, where $r=\sum_{i=1}^{S}C^i_{(n_0+1)r_1r_2}$.

Let us, assume that sequences (2) there are orthogonal sequences, i.e.

$$V^T\cdot V=diag\ [d_{1,1},...,d_{r,r}],\ d_{\alpha,\alpha}>0,\ \alpha=1,...,r\ ,\quad(8)$$

orthogonality conditions are satisfied, then solution to problem (4), (5) is found using solving a conditions Quadratic problem optimization

$$Y=V\cdot K,\quad J=(Y-Y_0)^T(Y-Y_0)\to\min\ ,\qquad(9)$$

where K is a r - dimensional vector. If there k_α and h_α are α -the component of the vectors K and H accordingly, then if $k_\alpha>0.5$, to $h_\alpha=1$, otherwise $h_\alpha=0$. And the solution to problem (9) is determined by the formula

$$K=(V^T\cdot V)^{-1}\cdot V^T\cdot Y_0\,.$$

If sequences (2) are not orthogonal, then the method used to solve the problem is attraction –based orthogonalization input sequences.

2. Solution of binary the synthesis problem for 4-Dimensional nonlinear modular dynamic system (4D-NMDS) with fixed memory n_0, limited communication $P=P_1\times P_2\times P_3$ and degree S are described in the form of two valued analogue of Volterra's polynomial is considered [2-4]:

The formula binary 4D-NMDS with fixed memory n_0 , limited communication $P=P_1\times P_2\times P_3$ and degree S are following:

$$y[n,c_1,c_2,c_3]=\sum_{i=1}^{S}\sum_{\nu=1}^{\lambda_i}\sum_{(\bar{j},\bar{\sigma},\bar{\rho})\in L_{i,\nu}}\sum_{\bar{\bar{\tau}}\in\Gamma_{i,\nu}}h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}]\times$$

$$\prod_{(\alpha,\beta,\gamma)\in Q_{i,\nu}}\prod_{\xi_{\alpha,\beta,\gamma}=1}^{m_{\alpha,\beta,\gamma}}\vartheta_{i,\nu}[n-\tau(\alpha,\beta,\gamma,\xi_{\alpha,\beta,\gamma}),c_1+\qquad(10)$$

$$+p_1(j_\alpha),c_2+p_2(\sigma_\beta),c_3+p_3(\rho_\gamma)],GF(2).$$

Here $n\in\{0,1,2,...\}$, $c_\alpha\in\{...,-1,0,1,...\},\alpha=\overline{1,3}$;

$$P_\alpha=\{p_\alpha(1),...,p_\alpha(r_\alpha)\},\ -\infty<p_\alpha(1)<...<p_\alpha(r_\alpha)<\infty,$$

$$p_\alpha(j)\in\{...,-1,0,1...\},\quad j=\overline{1,r_\alpha},\quad \alpha=\overline{1,3};$$

$$\lambda_i=|F(i)|\ ,$$

where

$$F(i)=\{(\ell_1,\ell_2,\ell_3,\overline{m})\big|\,\overline{m}=(m_{1,1,1},...,m_{1,1,\ell_3},m_{1,2,1},....,m_{1,2,\ell_3},...$$

$$...,m_{\ell_1,\ell_2,\ell_3}),\sum_{\alpha=1}^{\ell_1}\sum_{\beta=1}^{\ell_2}\sum_{\gamma=1}^{\ell_3}m_{\alpha,\beta,\gamma}=i;\quad m_{\alpha,\beta,\gamma}\in\{0,...,n_0+1\},$$

$$\alpha=\overline{1,\ell_1},\quad\beta=\overline{1,\ell_2},\gamma=\overline{1,\ell_3};$$

$$(\forall\alpha\in\{1,...,\ell_1\})(\exists\beta\in\{1,...,\ell_2\})(\exists\gamma\in\{1,...,\ell_3\})\Rightarrow(m_{\alpha,\beta,\gamma}\neq 0),$$
$$(\forall\beta\in\{1,...,\ell_2\})(\exists\alpha\in\{1,...,\ell_1\})(\exists\gamma\in\{1,...,\ell_3\})\Rightarrow(m_{\alpha,\beta,\gamma}\neq 0),$$
$$(\forall\gamma\in\{1,...,\ell_3\})(\exists\alpha\in\{1,...,\ell_1\})(\exists\beta\in\{1,...,\ell_2\})\Rightarrow(m_{\alpha,\beta,\gamma}\neq 0),$$
$$\ell_\sigma\in\{1,...,r_\sigma\},\sigma=\overline{1,3}\}$$
$$L(\ell)=L_1(\ell_1)\times L_2(\ell_2)\times L_3(\ell_3),$$
where
$$L_1(\ell_1)=\{\bar{j}=(j_1,...,j_{\ell_1})\big|1\le j_1<...,j_{\ell_1}\le r_1\},$$
$$L_2(\ell_2)=\{\bar{\sigma}=(\sigma_1,...,\sigma_{\ell_2})\big|1\le\sigma_1<...,\sigma_{\ell_2}\le r_2\},$$
$$L_3(\ell_3)=\{\bar{\rho}=(\rho_1,...,\rho_{\ell_3})\big|1\le\rho_1<...<\rho_{\ell_3}\le r_3\};$$
$$h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}]\equiv h_{i,(\ell_1,\ell_2,\ell_3,\overline{m})_\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}],$$
$$Q_{i,\nu}\equiv Q_0(i,(\ell_1,\ell_2,\ell_3,\overline{m})_\nu),$$
$$\Gamma_{i,\nu}\equiv\Gamma(\ell_1,\ell_2,\ell_3,\overline{m})_\nu,\ \ L_{i,\nu}\equiv L((\ell_1,\ell_2,\ell_3)_\nu),$$
where
$$\Gamma(\ell_1,\ell_2,\ell_3,\overline{m})=\underset{(\alpha,\beta,\gamma)\in Q_0(i,\ell_1,\ell_2,\ell_3,\overline{m})}{\times}\Gamma_1(m_{\alpha,\beta,\gamma}),$$
$$\Gamma_1(m_{\alpha,\beta,\gamma})=$$
$$=\{\bar{\tau}_{\alpha,\beta,\gamma}=(\tau(\alpha,\beta,\gamma,1),...,\tau(\alpha,\beta,\gamma,m_{\alpha,\beta,\gamma})):\ 0\le$$
$$\le\tau(\alpha,\beta,\gamma,1))<...<\tau(\alpha,\beta,\gamma,m_{\alpha,\beta,\gamma})\le n_0\};$$
$$Q_0(i,\ell_1,\ell_2,\ell_3,\overline{m})=$$
$$=\{(\alpha,\beta,\gamma):m_{\alpha,\beta,\gamma}\neq 0,\ \alpha=\overline{1,\ell_1},\ \beta=\overline{1,\ell_2},\ \gamma=\overline{1,\ell_3}\},$$
Let $n\in[0,\mathrm{N}]\equiv\{0,1,...\mathrm{N}\}$, $c_\alpha\in[0,C_\alpha]\equiv\{0,1,...,C_\alpha\}$, $\alpha=\overline{1,3}$ and the input $4D$-NMDS (10) receives following input
$$\{\vartheta_{i,\nu}[n,c_1,c_2,c_3]:n\in[0,N],\ c_\alpha\in[0,C_\alpha],\alpha=\overline{1,3}\},$$
$$\nu\in\{1,...,\lambda_i\},\ \ i\in\{1,...,S\} \tag{11}$$
and sequence (11) is an arbitrary binary sequence. The task of synthesis binary 4D- NMDS is to find such a value of impulse characteristics $h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}]$ for each $\bar{\bar{\tau}}\in\Gamma_{i,\nu}$, $(\bar{j},\bar{\sigma},\bar{\rho})\in L_{i,\nu}$, $\nu\in\{1,...,\lambda_i\}$, $i\in\{1,...,S\}$ at which the functional
$$J=\sum_{n=0}^{N}\sum_{c_1=1}^{C_1}\sum_{c_2=1}^{C_2}\sum_{c_3=1}^{C_3}(y[n,c_1,c_2,c_3]-y_0[n,c_1,c_2,c_3])^2 \tag{12}$$
is minimized.
$$\theta_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_k)=$$
$$=\{\prod_{(\alpha,\beta,\gamma)\in Q_{i,\nu}}\prod_{\xi_{\alpha,\beta,\gamma}=1}^{m_{\alpha,\beta,\gamma}}\vartheta_{i,\nu}[n-\tau^{(k)}(\alpha,\beta,\gamma,\xi_{\alpha,\beta,\gamma}),c_1+ \tag{13}$$
$$+p_1(j_\alpha),c_2+p_2(\sigma_\beta),c_3+p_3(\rho_\gamma)]\}.$$
In (13) each set
$$(n,c_1,c_2,c_3)\in[0,N]\times[0,C_1]\times[0,C_2]\times[0,C_3]$$
corresponds one line. Matrices are constructed sequentially:
$$\theta_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})=$$
$$=(\theta_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_1)\ ...\ \theta_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_{|\Gamma_{i,\nu}|}))$$
$$\theta_2(i,\nu)=(\theta_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_1)\ ...\ \theta_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_{|L_{i,\nu}|})), \tag{14}$$
$$\theta_3(i)=(\theta_2(i,1)\ ...\ \theta_2(i,\lambda_i)),\ \theta=(\theta_3(1)\ ...\ \theta_3(S))$$
And vectors
$$H_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})=$$
$$(h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_1],...,h_{i,\nu}[\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_{|\Gamma_{i,\nu}|}])^T, \tag{15}$$
$$H_2(i,\nu)=$$
$$=(H_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_1),...,H_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_{|D_{i,\nu}|}))^T,$$
$$H_3(i)=(H_2(i,1),...,H_2(i,\lambda_i))^T,$$
$$H=(H_3(1),...,H_3(S))^T.$$
Block matrix θ as ordinary matrix has dimension is
$$(N+1)(C_1+1)(C_2+1)(C_3+1)\times R$$
where $R=\sum_{i=1}^{S}C^i_{(n_0+1)r_1r_2r_3}$.

Block vector H is the same as a ordinary vector has following dimension:
$$(N+1)(C_1+1)(C_2+1)(C_3+1)\times 1.$$
Let
$$Y_0=(y_0[0,0,0,0],...,y_0[0,0,0,C_3],...,y_0[0,0,C_2,C_3],...$$
$$...,y_0[0,C_1,C_2,C_3],...,y_0[N,C_1,C_2,C_3])^T,$$
$$Y=(y[0,0,0,0],...,y[0,0,0,C_3],...,y[0,0,C_2,C_3],...$$
$$...,y[0,C_1,C_2,C_3],...,y[N,C_1,C_2,C_3])^T.$$
Then problem (10), (12) with the help of (14)-(15) has the following matrix-vector form:
$$Y=\theta\cdot H,\ GF(2), \tag{16}$$
$$J=(Y-Y_0)^T\cdot(Y-Y_0)\to\min \tag{17}$$
If the input sequence (11) is orthogonal, problem (16), (17) is solved similarly to problem (8), (9).

If the sequence (11) is not orthogonal, solve problem (16), (17) orthogonalization is first carried out for sequences (11).

3. The Condition of orthogonality for input sequences for binary 4D-NMDS.

Let
$$\{\upsilon_{i,\nu}[n,c_1,c_2,c_3]:n\in[0,N]\},\ c_\alpha\in[0,C_\alpha],\alpha=\overline{1,3},$$
$$\nu\in\{1,...,\lambda_i\},\ \ i\in\{1,...,S\}$$
are such that the matrix formed from them according to formulas (18) satisfies the condition orthogonality
$$V^T\cdot V=diag\{d_{1,1},...,d_{R,R}\};\ \ d_{\alpha,\alpha}>0,\ \alpha=1,...,R. \tag{18}$$
Then sequence (17) is called orthogonal input sequence (OIP) for binary 4D-NMDS (10).

Let $R_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})$, $R_2(i,\nu)$, $R_3(i)$, R are the number of columns according of the matrix $V_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})$, $V_2(i,\nu)$, $V_3(i)$.

It's clear that
$$R_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})=\left|\Gamma_{i,\nu}\right|,\quad R_2(i,\nu)=\left|\Gamma_{i,\nu}\right|\cdot\left|L_{i,\nu}\right|,$$
$$R_3(i)=\sum_{i=1}^{\lambda_i}R_2(i,\nu).$$
Theorem 1. Let the matrix be formed from input sequences (17) according to the formulas (14)-(15). In order to satisfy the condition orthogonality (18), is necessary and sufficient, so that it is fulfilled for everyone

$$V_2(i,\nu)^T V_2(i,\nu) =$$
$$= diag\{d_{1,1}(2,i,\nu),...,d_{R_2(i,\nu),R_2(i,\nu)}(2,i,\nu)\}, \qquad (19)$$
$$d_{\gamma,\gamma}(2,i,\nu) > 0, \ \gamma = 1,...,R_2(i,\nu),$$

where $d_{\gamma,\gamma}(2,i,\nu)$ is elements of matrix $V_2(i,\nu)^T V_2(i,\nu)$ and for all $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,...,S\}$, $\nu' \in \{1,...,\lambda_{i'}\}$, $i' \in \{1,...,S\}$, $(i,\nu) \neq (i',\nu')$ satisfied

$$V_2(i,\nu)^T V_2(i',\nu') = 0. \qquad (20)$$

Relations (19) are the condition of the own orthogonality of sequences (17), and correlation solution (8) – mutual orthogonality follower ties (17) and sequences

$$\{\upsilon_{i',\nu'}[n,c_1,c_2,c_3]: n \in [0,N]\}, \ c_\alpha \in [0,C_\alpha], \alpha = \overline{1,3},$$
$$\nu' \in \{1,...,\lambda_{i'}\}, \ i' \in \{1,...,S\}$$

Theorem 2. Let $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,..., S\}$. For roper orthogonality sequences

$$\{\upsilon_{i,\nu}[n,c_1,c_2,c_3]: n \in [0,N]\}, \ c_\alpha \in [0,C_\alpha], \alpha = \overline{1,3},$$

necessary and sufficient for everyone carried out so that for $(\bar{j},\bar{\sigma},\bar{\rho}) \in L_{i,\nu}$ carried out

$$V_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho}))^T \cdot V_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})) =$$
$$diag\{d_{1,1}(1),...,d_{R_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})),R_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho}))}(1)\},$$
$$d_{\alpha,\alpha}(1) > 0, \ \alpha = 1,...,R_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})),$$

where $d_{\alpha,\alpha}(1)$ is elements matrix

$V_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho}))^T \cdot V_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho}))$, and for all $(\bar{j},\bar{\sigma},\bar{\rho}) \in L_{i,\nu}$, $(\bar{j}',\bar{\sigma}',\bar{\rho}') \in L_{i,\nu}$, $(\bar{j},\bar{\sigma},\bar{\rho}) \neq (\bar{j}',\bar{\sigma}',\bar{\rho}')$

satisfies d $V_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho})^T \cdot V_1(i,\nu,\bar{j}',\bar{\sigma}',\bar{\rho}') = 0$.

Theorem 3. Let: I. For all $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,...,S\}$ sequences $\bar{\upsilon}_{i,\nu}[n,c_1,c_2,c_3]$ are sequences with period $T(i,\nu)+1$, $A_1(i,\nu)+1$, $A_2(i,\nu)+1$ and $A_3(i,\nu)+1$ according to the arguments n, c_1, c_2 and c_3 besides

$$\bar{V}_2(i,\nu)^T \bar{V}_2(i,\nu) =$$
$$= diag\{d_{1,1}(2,i,\nu),...,d_{R_2(i,\nu),R_2(i,\nu)}(2,i,\nu)\},$$
$$d_{\gamma,\gamma}(2,i,\nu) > 0, \ \gamma = 1,...,R_2(i,\nu),$$

where $d_{\gamma,\gamma}(2,i,\nu)$ is elements of $\bar{V}_2(i,\nu)^T \bar{V}_2(i,\nu)$, and matrix $\bar{V}_2(i,\nu)$ formed at sequences

$$\{\ \bar{\upsilon}_{i,\nu}[n,c_1,c_2,c_3]: \ n \in [0,T(i,\nu)]\},$$
$$c_1 \in [0,A_1(i,\nu)], c_2 \in [0,A_2(i,\nu)], c_3 \in [0,A_3(i,\nu)]$$

sequentially according to the formulas:

$$\bar{V}_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_k) =$$
$$[\prod_{(\alpha,\beta,\gamma)\in Q_{i,\nu}} \prod_{\xi_{\alpha,\beta,\gamma}=1}^{m_{\alpha,\beta,\gamma}} \bar{\upsilon}_{i,\nu}[n-\tau^{(k)}(\alpha,\beta,\gamma,\xi_{\alpha,\beta,\gamma}),c_1 +$$
$$p_1(j_\alpha), c_2 + p_2(\sigma_\beta), c_3 + p_3(\rho_\gamma)\ ]$$
$$\bar{V}_1(i,\nu,\bar{j},\bar{\sigma},\bar{\rho}) = (\bar{V}_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_1) \ ... \ \bar{V}_0(i,\nu,\bar{j},\bar{\sigma},\bar{\rho},\bar{\bar{\tau}}_{|\Gamma_{i,\nu}|})),$$
$$\bar{V}_2(i,\nu) = (\bar{V}_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_1) \ ... \ \bar{V}_1(i,\nu,(\bar{j},\bar{\sigma},\bar{\rho})_{|L_{i,\nu}|})).$$

II. a) For all $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,..., S\}$ and

$$(n,c_1,c_2,c_3) \in$$
$$\in [0,T(i,\nu)] \times [0,A_1(i,\nu)] \times [0,A_2(i,\nu)] \times [0,A_3(i,\nu)]$$

sequences $\upsilon'_{i,\nu}[n,c_1,c_2,c_3]$ determined by the formula:

$$\upsilon'_{i,\nu}[n,c_1,c_2,c_3] =$$
$$= \begin{cases} \bar{\upsilon}_{i,\nu}[n,c_1,c_2,c_3], & (n,c_1,c_2,c_3) \in F(i,\nu) \times (\underset{\alpha=1}{\overset{3}{\times}} G_\alpha(i,\nu)), \\ 0 & , \ (n,c_1,c_2,c_3) \notin F(i,\nu) \times (\underset{\alpha=1}{\overset{3}{\times}} G_\alpha(i,\nu)), \end{cases}$$

where

$$F(i,\nu) =$$
$$= [N_1(i,\nu) - \tau(i,\nu), N_1(i,\nu) - \tau(i,\nu) + T(i,\nu)] \subset [0,T'],$$
$$G_\alpha(i,\nu) =$$
$$= [D_\alpha(i,\nu), D_\alpha(i,\nu) + A_\alpha(i,\nu)] \subset [0,C'_\alpha], \alpha = \overline{1,3},$$
$$\tau(i,\nu) = \begin{cases} \max\{m_{1,1,1},...,m_{1,1,\ell_3}, m_{1,2,1},...,m_{1,2,\ell_3},...,m_{1,\ell_2,\ell_3},... \\ \qquad ...,m_{\ell_1,\ell_2,\ell_3}\} - 1, \ N_1(i,\nu) > 0, \\ 0 \qquad , \ N_1(i,\nu) = 0. \end{cases}$$

b) For all $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,...,S\}$ natural numbers $N_1(i,\nu)$, $D_1(i,\nu)$, $D_2(i,\nu)$, $D_3(i,\nu)$ and domain

$$[0,T'] \times [0,C'_1] \times [0,C'_2] \times [0,C'_3]$$

are such that for any $\nu' \in \{1,...,\lambda_{i'}\}$, $i' \in \{1,...,S\}$, where $\langle i,\nu \rangle \neq \langle i',\nu' \rangle$, the relations are satisfied $F(i,\nu) \cap F(i',\nu') = \varnothing$ or $G_\alpha(i,\nu) \cap G_\alpha(i',\nu') = \varnothing$, $\alpha = \overline{1,3}$; according to the arguments n, c_1, c_2 and c_3.

III. For all $\nu \in \{1,...,\lambda_i\}$, $i \in \{1,...,S\}$ sequence $\upsilon_{i,\nu}[n,c_1,c_2,c_3\ ]$ there is a periodic continuation of the sequence $\upsilon'_{i,\nu}[n,c_1,c_2,c_3]$ from the domain

$$[0,T'] \times [0,C'_1] \times [0,C'_2] \times [0,C'_3]$$

to other parts of the domain

$$[0,N] \times [0,C_1] \times [0,C_2] \times [0,C_3]$$

with periods $T(i,\nu)+1$, $A_1(i,\nu)+1$, $A_2(i,\nu)+1$ and $A_3(i,\nu)+1$ according to the arguments n, c_1, c_2 and c_3. Then the

$$\{\upsilon_{i,\nu}[n,c_1,c_2,c_3]: n \in [0,N]\}, \ c_\alpha \in [0,C_\alpha], \alpha = \overline{1,3},$$
$$\nu \in \{1,...,\lambda_i\}, \ i \in \{1,...,S\}$$

sequences are orthogonal input sequences for binary 4D-NMDS (10).

CONCLUSIONS

The paper considers the issue of solving the synthesis problem for 3D-NMDS and 4D-NMDS and obtaining a solution formula. In addition, the condition of orthogonality for input sequences for binary 4D-NMDS was received

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