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# Optimization of cargo deliveries under high and random demand

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**Abstract.** The aim of this paper is to further develop a stochastic model in terms of describing product volume demand with specific laws of distribution of continuous random variables and to develop a mathematical model for determining the optimal need for cargo deliveries under high and random demand.

***Keywords:*** *cargo delivery; transport problem; emergency situation; affected population; livelihood*

## 1. INTRODUCTION

A significant part of the logistics operations along the route of the material flow from the primary source of raw materials to the end consumer involves the use of various means of transportation. The costs of transport operations account for up to 50% of the total costs of logistics operations [1].

Transport logistics deals with the problems of technical and technological connectivity of transport process participants, coordination of their economic interests.

The tasks of transport logistics include: creating transport systems; ensuring the technological consistency of the transport and warehousing process, joint planning of the transport process, production and storage; determining the efficient delivery routes, etc.

One of the most important objectives of rescue units and groups during emergency response in peacetime is participation in providing the affected population with food, water, basic necessities, other material resources and services, living quarters for temporary accommodation, as well as providing first aid to the affected population. Analysis of the practice of rescue operations shows that one of the key challenges in ensuring livelihood of the affected population in the emergency area, in particular, delivery of essential humanitarian goods from storage depots, taking into account available capacities and resources, routes and time to complete the task [2].

An essential feature of inventory management is the uncertainty stemming from the inaccuracy or incompleteness of information on demand, supply, time delays of ordered goods, spoilage and other parameters of the logistics system, which requires finding an effective mechanism for inventory management in the face of such uncertainty. However, the published studies on this issue do not fully cover the approaches to solving this problem, being focused on the average expected deterministic indicators, which significantly reduces the reliability and validity of the results obtained on their basis.

In logistical practice, quite often there are situations where the average delivery time significantly exceeds the time during which the material resource (in what follows referred to as cargo) of one order is consumed. Increasing the size of the order to the volume that guarantees the reliable operation of production during the delivery time is usually impossible due to the limited capacity of the warehouse. Therefore, cargo is delivered in relatively small batches, multiples of the carrying capacity of the vehicle or cargo unit used. As a result, a significant portion of the shipment is in transit. The exact date of its arrival at the warehouse is unknown due to the randomness of the delivery time. This makes it very difficult to plan orders for a future period. Either the warehouse is overfilled, or there is a threat of production stoppage due to lack of cargo. The inertia of the system makes it challenging to keep the inventory at an optimal level that guarantees minimum costs.

In this paper, we consider the problem of selecting an efficient option for the use of capacities and resources of rescue centers of public agencies during cargo deliveries to temporary accommodation facilities in an emergency situation. The factors determining the specifics of the response of rescue centers in emergencies are shown. An approach to the selection of an efficient option for cargo delivery by a rescue unit on the basis of solving the transport problem is proposed. An efficient option for the implementation of the optimal plan of cargo delivery to the sites of temporary accommodation is proposed, taking into account the distribution of vehicles by deliveries and the implementation time of each delivery.

When solving the problems of optimization of deliveries (transportation) of various kinds of products, linear programming methods are most commonly used. Such methods are used, in particular, in solving transportation and allocation problems [3]. They have inherent deterministic nature of generating demand for delivery (transportation) and output of products. Mathematical methods for solving these problems are sufficiently well developed [4], [5]. At the same time, the construction of stochastic models that take into account the random nature of the generation of demand or random demand for products, and bringing them to a numerical solution for the deterministic nature of the output is a highly relevant problem [6].

## 2. CHOICE OF MODE OF TRANSPORT

The choice of mode of transport is addressed in conjunction with other logistics objectives, such as the creation and maintenance of optimal inventory levels, the choice of packaging, etc. The choice of the mode of transport optimal for a particular delivery is based on the information on the characteristic features of various modes of transport. In the following paragraphs, we consider the main logistically relevant advantages and disadvantages of road, rail, water and air transport [1], [7].

### Road transport

Traditionally used for short-distance transportation. Previously, short distances meant distances of 50-100 km. At present, because of the progress in the automobile industry and the development of the road network, distances of 200-300 km are considered short. Recently, however, distances of about 1,500 km are covered by heavy vehicles (trucks) with trailers.

One of the main advantages is high maneuverability. With road transport, cargo can be delivered "from door to door" with the required degree of urgency. This mode of transport ensures regularity of delivery, as well as the possibility of delivery in small batches. Compared with other modes of transport, the packaging requirements are less stringent.

The main disadvantage of road transport is the relatively high cost of transportation, the fee typically being set based on the maximum carrying capacity of the vehicle. Other disadvantages of this mode of transport include the urgency of unloading, the possibility of cargo theft and vehicle theft, and relatively low carrying capacity. Road transport is also environmentally unfriendly, which further hinders its use.

### Rail transport

This mode of transport is well suited for the transportation of various consignments under any weather conditions. Rail transport enables relatively fast delivery of cargo over long distances. At the same time, it makes it possible to organize loading and unloading operations effectively.

A significant advantage of the rail transport is a relatively low cost of cargo transportation, as well as the availability of discounted rates.

The disadvantages of rail transport are the limited number of carriers, as well as the low possibility of delivery to the points of consumption, i.e., in the absence of access roads rail transport has to be supplemented by road transport.

**Maritime transport**

This mode of transport accounts for the highest share of international freight traffic. Its main advantages are low freight rates and high carrying capacity.

The disadvantages of maritime transport are its low speed, strict packaging and securing requirements, and low frequency of shipments. Maritime transport depends heavily on weather and navigation conditions and requires a complex port infrastructure.

**Inland water transport**

Low freight rates. This mode of transport is the cheapest when transporting cargoes weighing more than 100 tons over a distance of more than 250 km.

The disadvantages of inland water transport, besides low delivery speed, include low geographical accessibility. This has to do with the limitations associated with the configuration of waterways, uneven depths and changing navigation conditions.

**Air transport**

The main advantages are the highest possible speed, the ability to reach remote areas, and high cargo safety.

The disadvantages include high freight rates and dependence on weather conditions, which reduces the reliability of meeting the delivery schedule.

**Pipeline transport**

Low prime costs and high capacity. The degree of cargo preservation by this mode of transport is high under any weather conditions.

The disadvantage of pipeline transport is the narrow range of cargoes to be transported (liquids, gases, emulsions).

**Problem statement**

We consider the problem of planning the volume of cargo supplies and deliveries between $m$ suppliers $(i = 1,\ 2,\ ...,\ m)$ and $n$ consumers $(j\ =\ 1,\ 2,\ ...,\ n)$. Suppose that $x_{ij}$ is the volume of products supplied by the $i$-th supplier to the $j$-th consumer under the condition that $x_{ij}$ is fulfilled at random demand $Y_J$ specified by a distribution function $S_i\left(y_j\right)$, and known quantities $\Psi_1$ and $\Psi_2$ describing the penalty for short-delivery and over-delivery of products in a unit volume, respectively. Note that on the basis of statistical processing of experimental data it is possible to establish an analytical expression (distribution law) of the distribution function $S$ written in general form and determine its parameters.

**Solution**

Suppose that the random demand in the $i$-th point is described by the density of the distribution of product volumes $f(y_j)$. Then the objective function can be written in the following form:

$$F(x)=\sum_{j=1}^{m}\left[\int_{x_j}^{\infty}\Psi_1(y_j-x_j)f_j(y_j)dy_j+\int_{0}^{x_j}\Psi_2(x_j-y_j)f_j(y_j)dy_j\right], \tag{1}$$

which has to be minimized. Note that a whole region can be chosen as the $j$-th point, where the demand for product volumes $Y_j$ is of a random nature.

The practice of processing statistical data on product volume demands shows that the normal distribution law can be chosen for continuous random volumes $Y_j$. We write the random demand for deliveries in the $j$-th point as a density

$$f_j(y)=\frac{1}{\sigma_j\sqrt{2\pi}}e^{-\frac{(y-m_j)^2}{2\sigma_j^2}}, \tag{2}$$

where $m_j$ is the mathematical expectation; $\sigma_j$ is the standard deviation.

Substituting the value of $f_j(y_j)$ from (2) into (1), we obtain

$$\int_{\alpha}^{\beta}\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{(m-x)^2}{2\sigma^2}}dx=\Phi\left(\frac{\beta-m}{\sigma}\right)-\Phi\left(\frac{\alpha-m}{\sigma}\right). \tag{3}$$

It is known that

$$\int_{\alpha}^{\beta}\frac{x}{\sigma\sqrt{2\pi}}e^{-\frac{(m-x)^2}{2\sigma^2}}dx=m\left[\Phi\left(\frac{\beta-m}{\sigma}\right)-\Phi\left(\frac{\alpha-m}{\sigma}\right)\right]-\frac{\sigma}{\sqrt{2\pi}}\left[e^{-\frac{1}{2}\left(\frac{\beta-m}{\sigma}\right)^2}-e^{-\frac{1}{2}\left(\frac{\alpha-m}{\sigma}\right)^2}\right] \tag{4}$$

where $\Phi(\bullet)$ is a Laplace function.

Performing the appropriate transforms, we obtain an analytical expression for the objective function

$$F(x)=\sum_{j=1}^{m}\left(m_j-x_j\right)\left[\Psi_1-\left(\Psi_1+\Psi_2\right)\Phi\left(\frac{x_j-m_j}{\sigma_j}\right)+\Psi_2\Phi\left(-\frac{m_j}{\sigma_j}\right)\right]+$$
$$+\frac{\sigma_j}{\sqrt{2\pi}}\left\{\left(\Psi_1+\Psi_2\right)\exp\left[-\frac{1}{2}\left(\frac{x_j-m_j}{\sigma_j}\right)^2\right]-\Psi_2\exp\left[-\frac{1}{2}\left(-\frac{m_j}{\sigma_j}\right)^2\right]\right\}, \tag{5}$$

where $\Psi_1$, $\Psi_2$ – are the penalties per unit of short-delivered and overdelivered products, respectively; $j=1,\ 2,\ \ldots,\ m$ is the number of points of consumption.

Objective function (5) allows determining optimal values of volumes of deliveries $x_{ij}$ for each consumer in the presence of constraints on the total volume of deliveries, i.e., $\sum_{j=1}^{m}x_j\le a$. Note that for the case $j=1$ (one supplier and one consumer) and with unlimited volume of deliveries, the optimal values of $x_j$ for the objective function $F(x)$ can be determined from (5). Thus, for a random demand

$f(y)$ with a mathematical expectation $m = 2$ and a standard deviation $\sigma = 1$ at $\Psi_1 = 5, \Psi_2 = 1$ for the selected values $x = 0,1,2,3,4;$ $F(0) = 5.4;$ $F(1) = 2.6;$ $F(2) = 2.04;$ $F(3) = 1.52;$ $F(4) = 2.28.$

The objective function reaches the minimum value at $x = 3.$ Note that the average volume of deliveries is $m = 2.$ For a more general case (with several consumers and known parameters of random demand $m_j$ and $\sigma_j$) the optimal values of product consumption $x_j$ can be determined by brute force or sequential analysis of variants, or by the dynamic programming method. To simplify the calculations, by choosing for each consumption point a step $\Delta x = 1,$ in the interval $0 < x_j < x$ the optimal value of $x_j$ can be found by the dynamic programming method.

## Conclusion

We have obtained mathematical model (5), which allows determining optimal values of product volumes for any number of consumers $(j = 1,\ 2,\ ...)$ with known numerical characteristics of normally distributed random product demand. A stochastic model for determining the optimal volumes of deliveries from $m$ enterprises to $n$ consumers under a clear law of distribution of consumption volumes has been proposed. The dynamic programming method has been used to obtain the solution.

The proposed model makes it possible to predict with a high degree of reliability the arrival of material resources at the warehouse at long and random delivery times. The optimal order schedule is formed on the basis of the inventory forecast. The optimality criterion is the minimum total cost of storing an inventory of material resources. The constraints are critical inventory values (minimum and maximum). The model allows for optimization of the delivery schedule by other criteria as well.

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