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Model of queuing-inventory system with catastrophes and random replenishment policy

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Abstract. A model of a queuing-inventory system with catastrophes and a random replenishment policy is developed in the form of a one-dimensional Markov chain. Explicit formulas to calculate the steady-state probabilities of the constructed Markov chain are obtained and performance measures of the investigated system are determined by using these formulas. Results of numerical experiments are demonstrated.

Keywords: *queuing-inventory system; catastrophes; randomized replenishment policy; Markov chain; calculation algorithm*

1. INTRODUCTION

In the classic models of inventory management systems (IMS), it is assumed that the period to sell the inventory to demands is equal to zero. Another assumption in the research conducted in this area is that system inventories are durable, long-lasting, that is, they never perish. However, in real life, there are systems whose inventory perish or are destroyed as a result of certain events. Systems of this type are divided into two classes. In the first class of IMS, inventory perish at a certain time interval. Such systems have been widely studied in recent literature; see [1-7]. Along with systems of this type, there are also systems whose inventory is instantly destroyed as a result of a catastrophe. Systems whose inventory can be destroyed one by one are studied in articles [8-11]. However, models of systems whose whole inventory can be destroyed suddenly as a result of any kind of catastrophes have been underexplored so far [12]. In the model of IMS studied in the latterpaper, it was assumed that the "Up to S" replenishment policyis used. In other words, when the inventory level reaches a certain level of s, $0 < s < S$, an order is made, and when inventory arrive, the inventory level reaches the full capacity of the warehouse. In this paper a similar model using a randomized replenishment policy is considered.

2. DESCRIPTION OF THE MODEL AND PROBLEM STATEMENT

The demands in the considered IMS form a Poisson flow with rateλ. The system features self-service, which means that the demand servicing time is zero. Each demand requires a unit size of inventory. If the inventory level is zero at the moment of arrival of any demand, then it is lost. Catastrophes may occur in the system and it is assumed that the flows of these catastrophes are also Poisson flows with rateκ . As a result of catastrophes, all inventory is instantly destroyed, i.e., the inventory level of the system drops to zero. If the inventory level of the system is zero, then catastrophes do not affect the operation of the system.
To increase the inventory level of the system, the randomized replenishment policy is used. This policy is defined as follows: the inventory is reordered when the inventory level of the system drops to zero. In this case, the volume of the order is a random quantity, that is, the probability that the volume of the

order is equal to m is α_m, so $\sum_{m=1}^{S}\alpha_m = 1, \alpha_s > 0$. The lead time is a random quantity and has exponential distribution with rateν.

The aim of the study is to find the stationary distribution of the described model and based on it to calculate the main performance measures of the system. The main performance measures of the system are following: the average inventory level (S_{av}), the average reorder rate (RR), the average volume of the incoming inventory (V_{av}) and the loss probability of demands (P_l) .

3. SOLUTION

The operation of the system can be described by a one-dimensional Markov chain (MC). Suppose that the state m of this MC indicates the inventory level, $m = 0,1,\dots,S$, where S is the maximum size of system storage. In other words, the state space of this MC is given by the set $E = \{0,1,\dots,S\}$.

First, we look at the issue of constructing the generator of the Markov chain. The transition from any state $m$to state $m'$is denoted by $m \to m'$. Considering the adopted replenishment policy, demand servicing and consequences of a catastrophe, the diagram of states of the system is as shown in Fig. 1

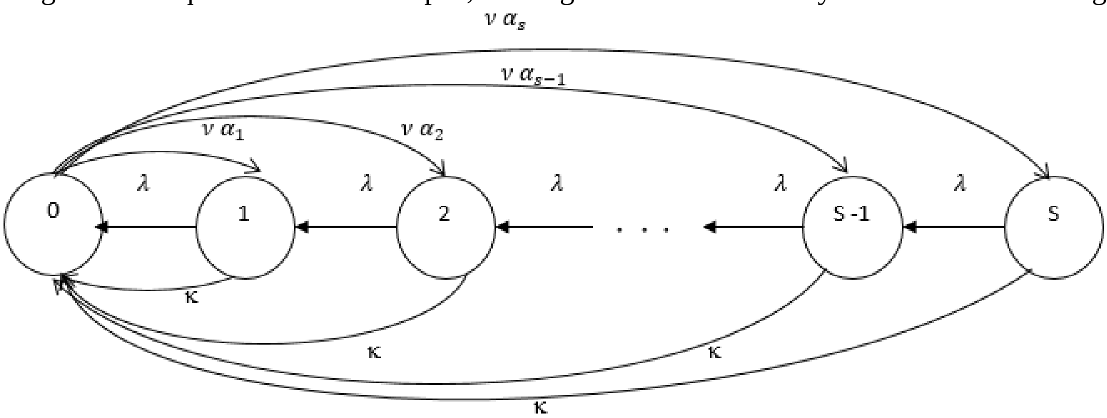


Fig. 1. Diagram of states of the system

- When $m > 0$, the rate of transition $m \to m-1$ is λ;
- When $m > 0$, the rate of transition $m \to 0$ is κ;
- The rate of transition $0 \to m$ is $\nu_m = \nu\,\alpha_m$.

After constructing the generator of the considered MC, the system of equilibrium equations for the stationary probabilities of the states is written as follows:

$\nu p(0) = p(1) + \kappa\sum_{m=1}^{S} p(m)$; (1)

$(\lambda+\kappa)p(m) = \lambda p(m+1) + \nu\alpha_m p(0), 1 \le m \le S-1$ (2)

$(\lambda+\kappa)\,p(m) = \nu\alpha_s p(0)$; (3)

The normalizing condition is added to equations (1)-(3):

$\sum_{m=0}^{S} p(m) = 1$ (4)

Solving equations (1)-(4) recursively, we get the following formulas:

$p(m) = r_m p(0), \quad 0 \le m \le S,$ (5)

where $p(0) = \left(\sum_{m=0}^{S} r_m\right)^{-1}$; (6)

$r_0=1$

$r_m = \frac{1}{\lambda+\kappa}(\lambda\, r_{m+1} + \nu_m), 1 \le m \le S-1$ (7)

$r_s = \frac{\nu}{\lambda+\kappa}$. (8)

After determining the steady-state probabilities, the performance measures of the system are calculated as follows. The average inventorylevelis defined as the mathematical expectation of theconsidereddistribution, i.e.,

$$S_{av} = \sum_{m=1}^{S} \mathrm{m}p(\mathrm{m}) \qquad (9)$$

The average rate of orders made to replenish the inventory(i.e. Reorder Rate, RR)

$$RR = \kappa(1 - p(0) + \lambda p(1)) \qquad (10)$$

The average volume of incoming inventory (i.e. average order size)

$$V_{av} = p(0)\sum_{m=1}^{S} \mathrm{m}\alpha_{\mathrm{m}} \qquad (11)$$

If at the moment of arrival of demands, the inventory level is zero, demands leave the system without inventory. For this reason, loss probability of demandsis calculated as follows:

$$P_l = p(0) \qquad (12)$$

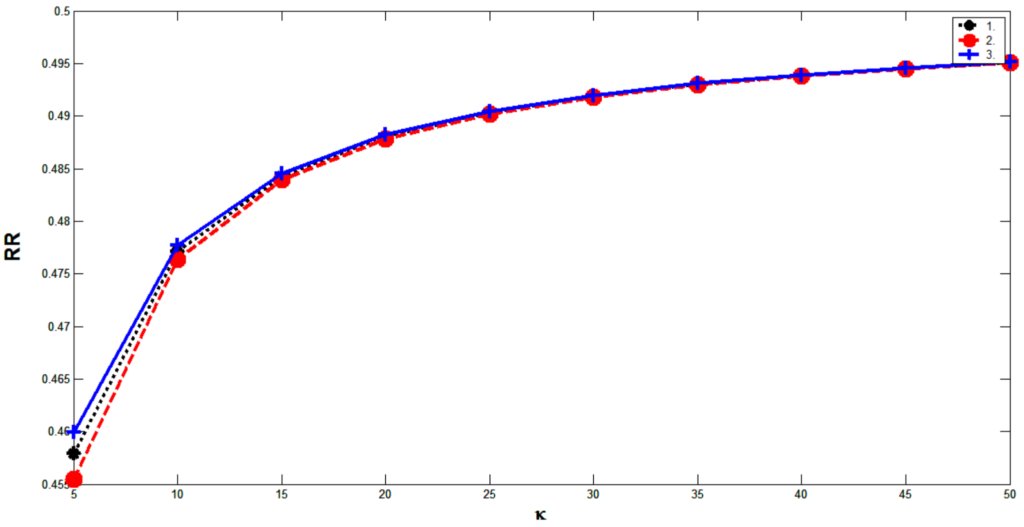
Thus, the performance measures of the system are calculated by using formulas (9)-(12).

4. NUMERICAL RESULTS

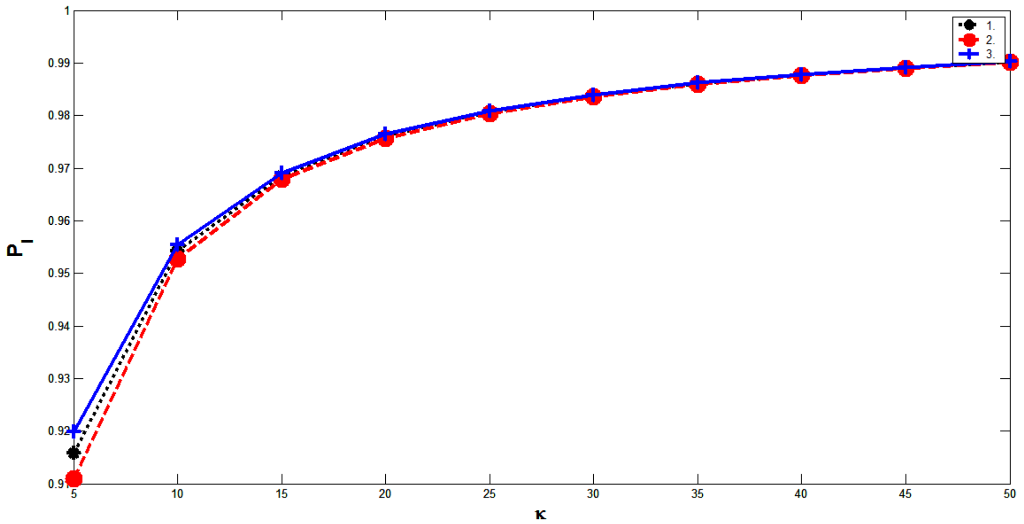
Belowwe consider the results of numerical experiments carried out to calculate the performance measures of the system using the obtained formulas. The purpose of these experiments is to study the relationship between the performance measures of the system and its initial parameters.

In all experiments, the storage size of the system is assumed to be constant, i.e., we assume that $S = 50$. Three schemes of variation of the parameters α_m are considered: 1) all these parameters are assumed to be equal to each other, i.e., they are constant and $\alpha_m = \frac{1}{50}$; 2) the parameters α_m are quantities that increase by the argument m; i.e., $\alpha_\kappa \leq \alpha_{\kappa+1}$ 3)the parameters α_m are quantities that decrease by the argument m; i.e., $\alpha_\kappa \geq \alpha_{\kappa+1}$, $\kappa = 1,2, \ldots, S-1$. The values of other initial parameters are shown in the corresponding graphs.

Fig. 2 shows the relationship between the performance measures of the system and the catastrophe rate. The graphs show that when the value of the parameter κ increases, the average reorder rate (RR) increases as well, see Fig. 2(a). This is due to the fact that when the parameter κ increases (i.e., when the catastrophe rate increases), the inventory of the system rapidly drops to zero, and therefore the reorder rate increases. In this case the loss probability of demands (P_l) also increases. See Fig. 2(b), because the system inventory rapidly drops to zero, see Fig. 2(b). As expected, as the catastrophe rate increases, the average inventory level of the system decreases, see Fig. 2(c), but the average volume of incoming inventory (V_{av}) increases, see Fig. 2(d). Note that the scheme of variation of the probabilities α_m has no significant effect on the measures $RR$and P_l, but has significant effect on the other measures.In the second scheme, the values of the functions S_{av}, V_{av} are greater than in the third scheme. Interestingly, scheme (1) is between the values of schemes (2) and (3). This is because the second scheme has a high probability of a large amount of incoming inventory.


RR
κ
1.
2.
3.
0.5
0.495
0.49
0.485
0.48
0.475
0.47
0.465
0.46
0.455
5
10
15
20
25
30
35
40
45
50

(a)


P_l
κ
1.
2.
3.
1
0.99
0.98
0.97
0.96
0.95
0.94
0.93
0.92
0.91
5
10
15
20
25
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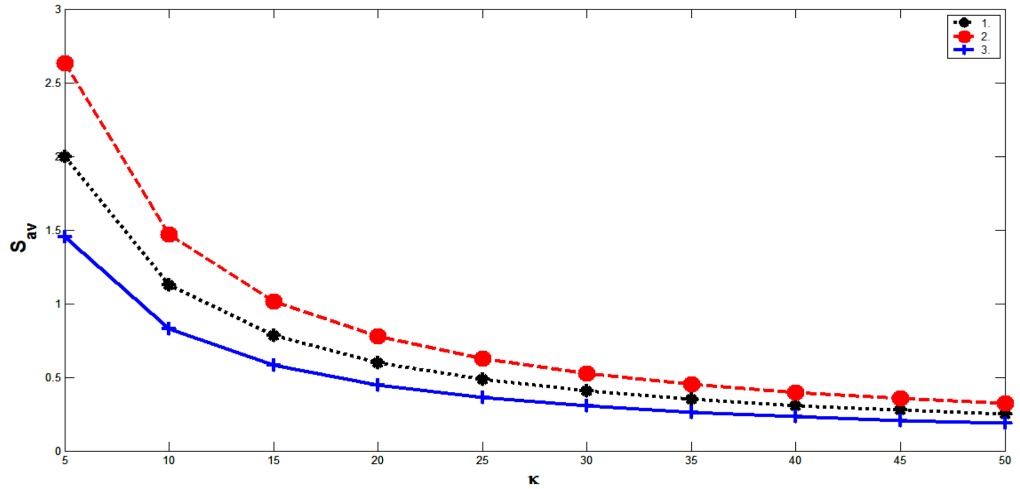
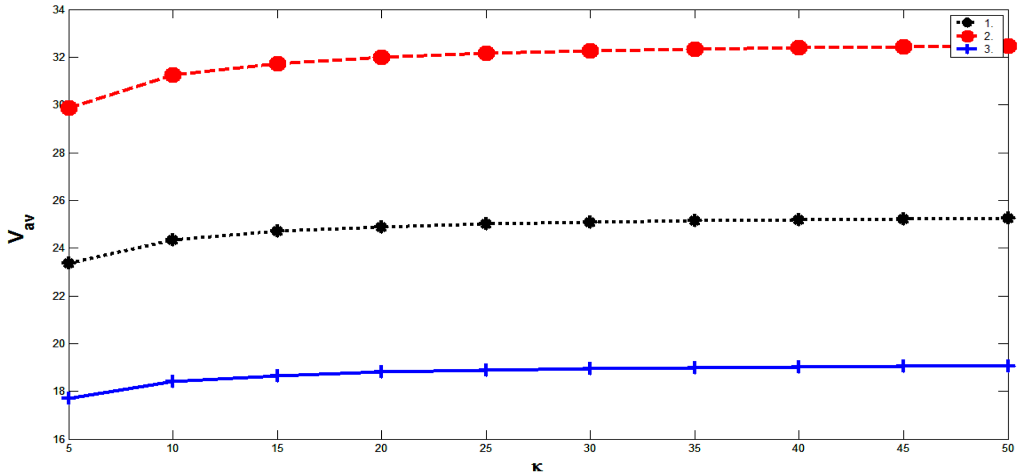
35
40
45
50

(b)



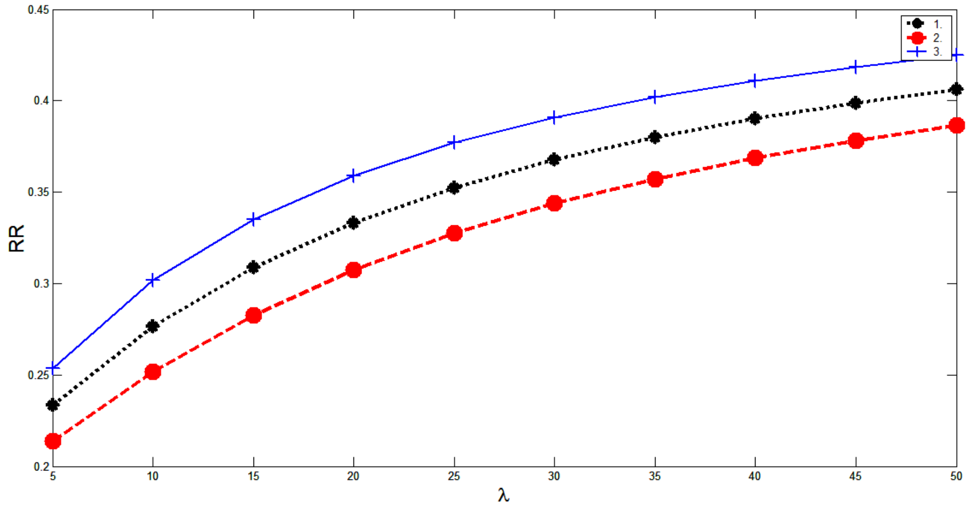
(c)



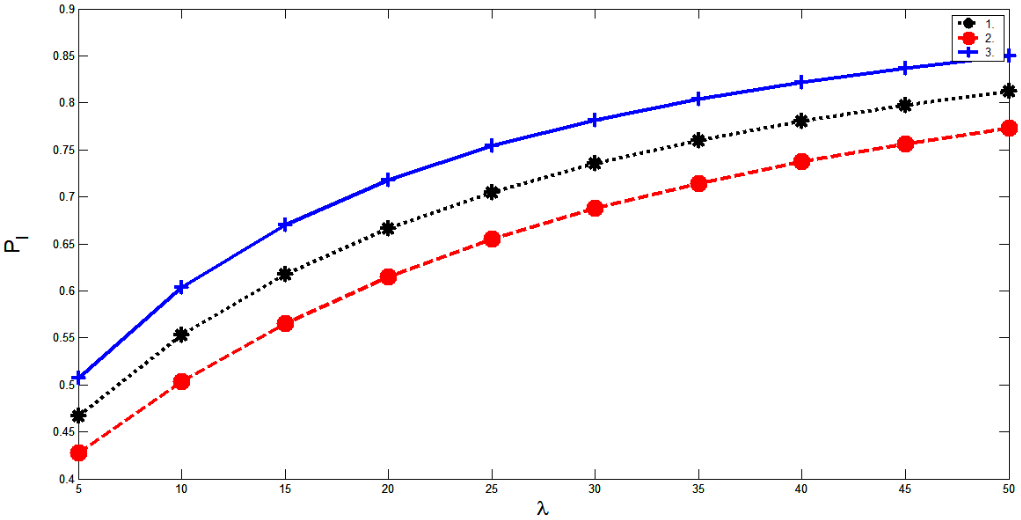
(d)

Fig.2. Performance measuresvsκ; $\lambda = 20, \upsilon = 0{,}3$.

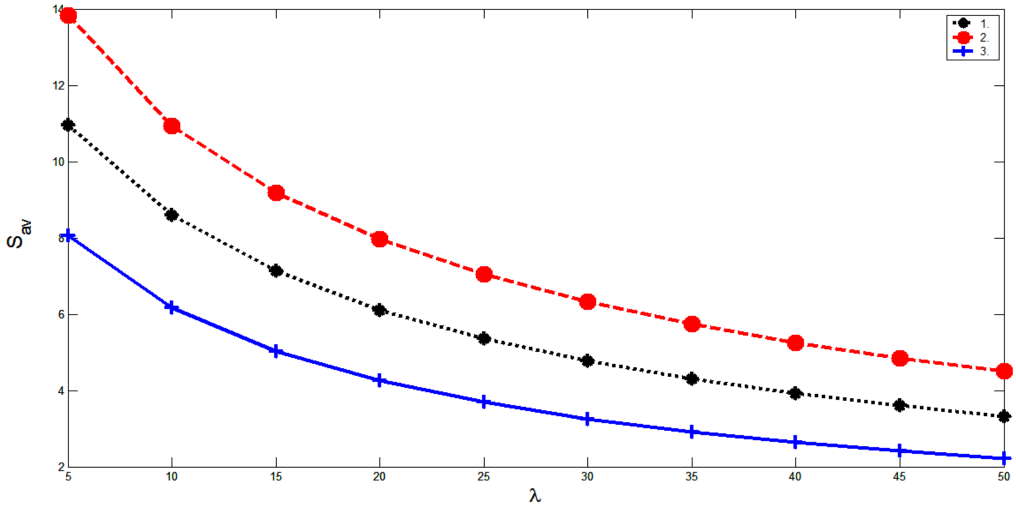
Fig. 3 shows the relationship between the performance measures of the system and the demand rate. The graphs show that when the value of the parameter λ increases, the average reorder rate increases as well, see Fig. 3(a). This is due to the fact that when the parameter λ increases, the inventory of the system rapidly drops to zero, and therefore the average reorder rate increases. When the demand rate increases, the loss probability of demands (P_l) also increases, see Fig. 3(b), because the system inventory rapidly drops to zero, the average inventory level of the system (S_{av}) decreases, see Fig. 3(c), and at the same time the average volume of incoming inventoryincreases, see Fig. 3(d). Unlike Fig. 2, herethe schemasof variation of probabilities α_m has significant effect on all performance measures. In the measures RR and $P_l$vs parameter λ relationship, the values of the measures according to scheme (2) for determining the parameters α_m are smaller than those according to scheme (3), and, as expected, scheme (1) is between the of schemes (2) and (3). However, in the measures S_{av} ,V_{av}vs parameter λ relationship, the values of the measures for determining the parameters $\alpha_m$in scheme (2) are greater than the values in scheme (3).



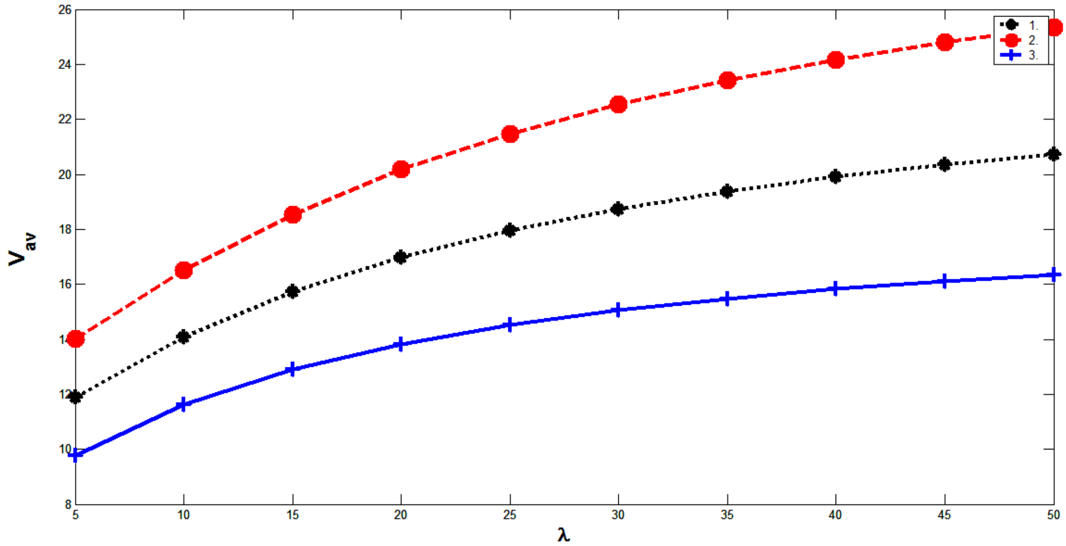
(a)



(b)



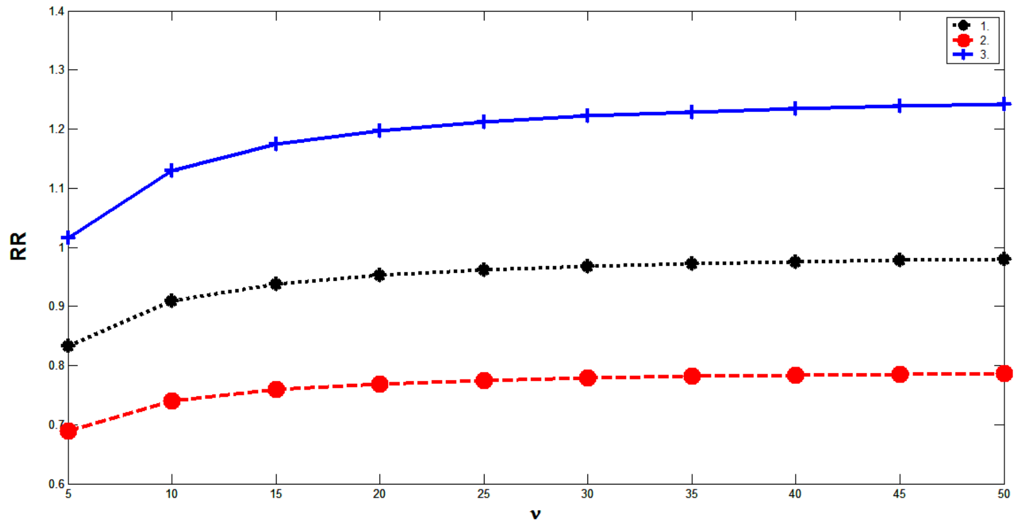
(c)



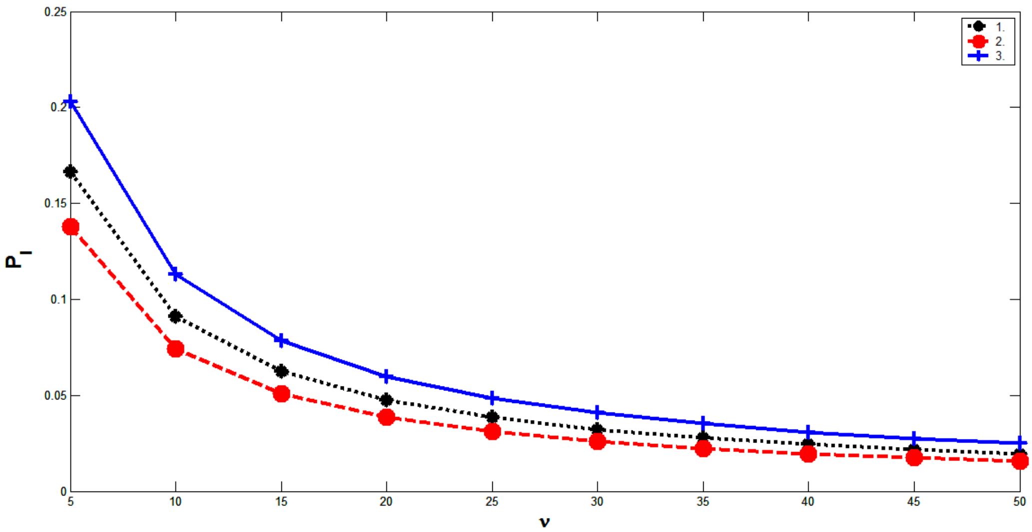
(d)

Fig.3.Performance measures vs λ; $\kappa = 0{,}3, \upsilon = 0{,}5$.

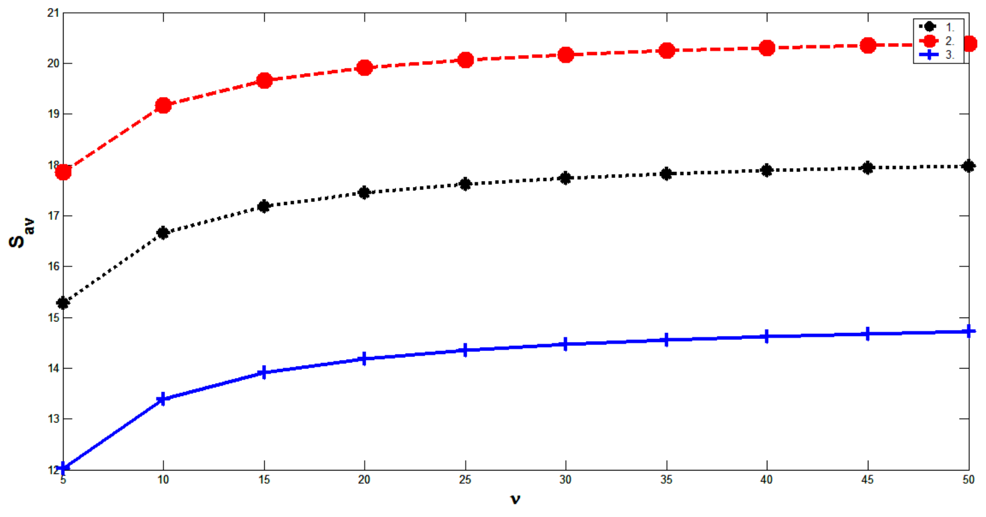
Fig. 4 shows the relationship between the performance measures of the system and the replenishment rate. At first glance, it seems that when the value of this parameter increases, the average reorder rate should decrease. However, we can see from the graphs that when the value of this parameter increases, the average reorder rate also increases, see Fig. 4(a). Then, when the parameters α_m increase,the probability of the system inventory dropping to zero decreases, in other words, the probability of the system having stocks increases. We can see from formula (10) that in this case,the average reorder rate increases as well. The decrease in the loss probability of demands is due to the fact that when the parameter ν increases, the probability of the system inventory dropping to zero decreases, see Fig. 4(b). When the rate of replenishment increases, the average inventory level increases, see Fig. 4(c),but the average volume of incoming inventory decreases, see Fig. 4(d). The scheme of variation of the parameters $\alpha_m$has no significant effect on the average volume of incoming inventory, but has significant effect on the other measures. Thus, in the RR and P_l vs ν relationship graph, the value of the measure in the 3rd scheme for determining the parameters α_m is greater than the value of the measure in the 2nd scheme, because the average inventory level increases slowly. In the average inventory level relationship graph, in the 3rd scheme for determining the parameterα, this measure takes on smaller values than in the 2nd scheme. As expected, in all the graphs, scheme (1) for determining the parameters α_m is between the of schemes (2) and (3).



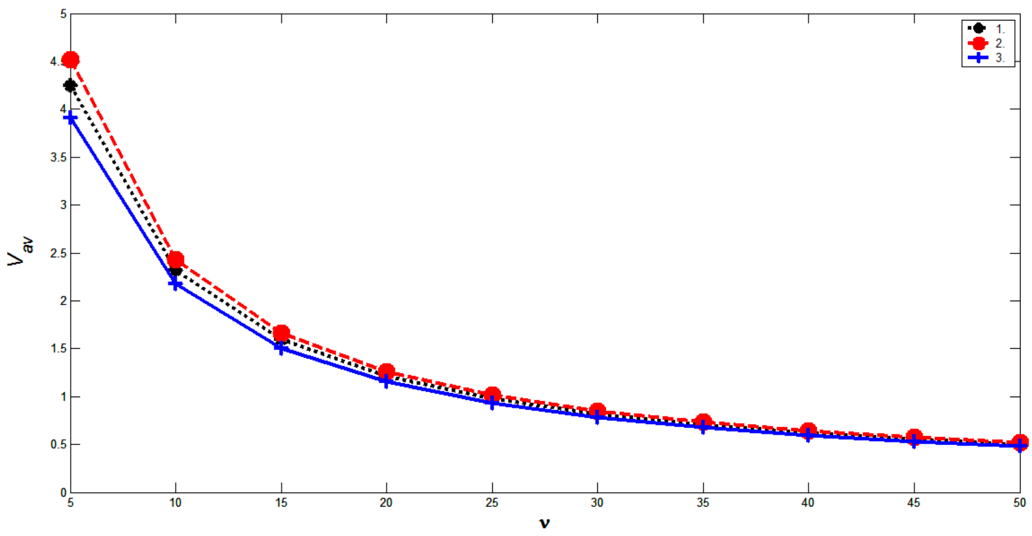
(a)



(b)



(c)



(d)

Fig.4. Performance measuresvsν; $\lambda = 20, \kappa = 0{,}3$.

5. CONCLUSIONS

A queuing-inventory model has been developed in which inventory can be instantly destroyed as catastrophes occur. Randomized replenishment policy is used, i.e., the size of order is a random quantity. Mathematical model of the investigated systems is a one-dimensional Markov chain with finite state space. Explicit formulas are obtained to calculate the steady-state probabilities of the constructed Markov chain. Based on the developed formulas, performance measures of the investigated QIS are determined and results of numerical experiments are demonstrated.

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