2nd International Conference on Problems of Logistics, Management and Operation in The East-West Transport Corridor (PLMO 2023) Baku, Azerbaijan
DOI:10.54381/plmo2023.27

Determination the segments in pipelines of complex structure containing leakage

Kamil AIDA-ZADE
Institute of Control Systems, Baku, Azerbaijan
Yegana ASHRAFOVA*
Baku State University, Institute of Control Systems, Baku, Azerbaijan
**Corresponding author*. E-mail: ashrafov.yegana@gmail.com

Abstract. The paper proposes a numerical solution to the problem of determining segments of the hydraulic network that are suspicious for the presence of a possible leakage, if there are additional observations of the regimes of unsteady fluid flow at any points in the pipeline. The problem is reduced to a problem of parametric optimal control without initial conditions with unseparated boundary conditions, for the solution of which first-order optimization methods are used. The results of numerical experiments are carried out.

Keywords: *pipeline of complex structure; segments containing leakages; leakages; optimization methods*

INTRODUCTION

A serious problem concerning the transportation of raw materials through pipelines is timely detection of leaks and their locations. To monitor the state of pipeline segments, much attention is given to the creation of methods and devices for leak detection. For many pipeline networks for which visual inspection and the application of such devices present difficulties, leak locations can be determined by applying mathematical methods with the use of state characteristics measured at certain pipeline points (see [1–4]).

This paper differs from many other studies in which leak locations and amounts were determined either in a steady flow regime in a pipeline of complex structure or in a transient flow regime in a pipeline consisting of a single linear segment. In this study, we numerically solve an inverse problem of determining leak locations and amounts in an unsteady flow in a pipeline network of complex (loopback) structure. The problem is described by a system made up of numerous subsystems of two hyperbolic partial differential equations with impulse actions specified at possible leakage points on pipeline network segments.

Another feature of the problem is the assumption that, due to the long duration of the process under study, exact information on its initial state is not available at the time of monitoring and that the states of the process (which is distributed in space) cannot be quickly measured at all points. Instead, there is information on a variety of possible initial states of the process and some state (regime) characteristics are measured at certain pipeline points starting at this time. One more feature of the problem is that its boundary conditions are specified as nonseparated relations (determined by physical laws) between the states at the endpoints of adjacent pipeline segments.

The considered inverse problem in variational formulation belongs to the class of optimal control problems for distributed parameter systems. It is solved numerically by applying first-order optimal control methods. Accordingly, we derive formulas for the components of the cost functional gradient

with respect to the parameters to be identified. Numerical results for test inverse problems in the presence of measurements with various levels of errors in the input data are presented.
Note that the problem considered in this work and the approach employed for its numerical solution can be used, for example, to determine the locations and strengths of oscillation sources in complicated multistage mechanical systems, external actions on the ecological state of interrelated zones (regions), and in other applications described by partial differential equations of other types and forms.

1. PROBLEM STATEMENT

Let the unsteady regime of fluid flow in a pipeline network of a complex looped structure containing M segments and N vertices be laminar. The set of segments we denote by K: $K=\{(k_i,k_j):k_i,k_j\in I\}$, where I – is a set of vertices; $l^{k_ik_j}$, $d^{k_ik_j}$ – respectively length and diameter of (k_i,k_j)–th segment, $(k_i,k_j)\in K$.

Let it be known that in some segments (k_{i_s},k_{j_s}) of the network, $s=1,...,Z,$ on unknown points $\xi^{k_{i_s},k_{j_s}}\in(0;l^{k_{i_s},k_{j_s}})$ there are leakages, the volume of which are determined by the functions $q^{k_{i_s},k_{j_s}}(t)$. The set of such segments with the quantity Z we denote by $K^{loss}=\{(k_{i_1},k_{j_1}),...,(k_{i_Z},k_{j_Z})\}\subset K,$ $\xi=(\xi^{k_{i_1},k_{j_1}},...,\xi^{k_{i_1},k_{j_1}}),$ $q^{loss}(t)=(q^{k_{i_1},k_{j_1}}(t),...,q^{k_{i_1},k_{j_1}}(t)).$

The problem is to find the segments of the hydraulic network that are suspicious for the presence of a possible leak using the mathematical model below and the observed information [1,2].

In order to solve the problem, we consider a functional that determines the magnitude of the deviation of the observed modes at given points of the oil pipeline segment from the calculated ones:

$$J(\xi,q^{loss})=\frac{1}{\nu}\sum_{j=1}^{\nu}\left[\Phi(\xi,q^{loss};\hat{Q}_j,\hat{P}_j)+\varepsilon_1\left\|q(t)-\hat{q}\right\|^2_{L_2^Z[t_0,T]}+\varepsilon_2\left\|\xi-\hat{\xi}\right\|^2_{R^Z}\right]\to\min, \tag{1}$$

$$\Phi(\xi,q^{loss};\hat{Q}_j,\hat{P}_j)=\sum_{m\in\tilde{I}_q^f}\int_{\tau}^{T}[Q^m(t;\xi,q^{loss}(t),\hat{Q}_j,\hat{P}_j)-Q^m_{mes}(t)]^2\,dt,$$

where $Q^m(t;\xi,q^{loss}(t),\hat{Q},\hat{P}),m\in I_q^f$ – are the calculated values of the flow rate at the observed points as a result of solving the boundary value problem under any possible initial conditions $\hat{Q},\hat{P}$: $\hat{Q}=\{\hat{Q}_1^{ks}(x),\hat{Q}_2^{ks}(x),...,\hat{Q}_\nu^{ks}(x)\}_{(k,s)\in K}$, $\hat{P}=\{\hat{P}_1^{ks}(x),\hat{P}_2^{ks}(x),...,\hat{P}_\nu^{ks}(x)\}_{(k,s)\in K}$ and for given allowable places and volumes of leaks $(\xi,q^{loss}(t))$; $[\tau,T]$– time interval for tracking the process, the modes of which no longer depend on the initial conditions; $\hat{\xi},\hat{q}\in R^m,\varepsilon_1,\varepsilon_2$ – are the regularization parameters. Since the initial conditions determined at the moment of time t_0 do not affect the process in the interval $[\tau,T]$, the exact knowledge of the initial value t_0 and initial functions $\hat{Q}_0^{ks}(x),\ \hat{P}_0^{ks}(x),\ (k,s)\in J$ on the value of the functional (1) is not important [5].

The process of unsteady isothermal laminar flow of a droplet liquid with a constant density ρ on the (k,s)-th linear segment of the oil pipeline network can be described by the following system of two linear differential equations of hyperbolic type [3,6]:

$$-\frac{\partial P^{ks}(x,t)}{\partial x}=\frac{\rho}{S^{ks}}\frac{\partial Q^{ks}(x,t)}{\partial t}+2a^{ks}\frac{\rho}{S^{ks}}Q^{ks}(x,t),$$

$$-\frac{\partial P^{ks}(x,t)}{\partial t}=\begin{cases}c^2\dfrac{\rho}{S^{ks}}\left(\dfrac{\partial Q^{ks}(x,t)}{\partial x}+q^{ks}(t)\delta(x-\xi^{ks})\right), & x\in(0,l^{ks}),(k,s)\in K^{loss},\\[2ex] c^2\dfrac{\rho}{S^{ks}}\dfrac{\partial Q^{ks}(x,t)}{\partial x}, & x\in(0,l^{ks}),(k,s)\notin K^{loss}.\end{cases} \tag{2}$$

here: $t \in (t_0, T]$; $P^{ks}(x,t), Q^{ks}(x,t)$ –are the pressure and flow rate, respectively, at the time t in the point $x \in (0, l^{ks})$ of the segment (k,s) of the pipeline network; c – is the sound velocity in the fluid; S^{ks} – is the area of an internal cross-section of the segment (k,s) –; a^{ks} – is the dissipation coefficient on the sgment (k,s) (we may consider that the kinematic coefficient of viscosity γ is independent of the pressure and the condition $2a^{ks} = 32\gamma/(d^{ks})^2 = const$) is quite accurate for a laminar flow); $\delta(\cdot)$ – is the Dirac function.

For system (2), $2M$ -number boundary conditions must be specified. For internal nodes of the network, we use the material balance conditions

$$\sum_{s \in I_k^+} Q^{ks}(l^{ks}, t) - \sum_{s \in I_k^-} Q^{ks}(0,t) = \tilde{q}^k(t), \quad k \in I^{\text{int}}, \quad t \in [t_0, T], \tag{3}$$

and flow continuity:

$$P^{k_i k}(l^{k_i k}, t) = P^{k k_j}(0,t), \qquad k_i \in I_k^+, \; k_j \in I_k^-, \; k \in I^{\text{int}}, \; t \in [t_0, T], \tag{4}$$

where $\tilde{q}^k(t)$ – is the given external inflow or outflow $(\tilde{q}^k(t) > 0)$ for that internal k – vertex; I_k^+ – is the set of vertex numbers from which segments lead to k – vertex, I_k^- – is the set of vertex numbers to which segments lead from the k – vertex.

At each vertex that is not an internal vertex of the network, i.e. from I^f, we set the pressure value (we denote the set of such vertices by $I_p^f \subset I^f$):

$$\begin{cases} P^{ns}(0,t) = \tilde{P}^n(t), & s \in I_n^+, \; if \;\; I_n^- = \varnothing, \\ P^{sn}(l^{sn}, t) = \tilde{P}^n(t), & s \in I_n^-, \; if \;\; I_n^+ = \varnothing, \end{cases} \qquad n \in I_p^f, \; t \in [t_0, T], \tag{5}$$

Here $\tilde{Q}^m(t)$, $\tilde{P}^n(t)$, $n \in I_p^f$ – are given functions that determine the modes of operation of the sources. Note that the boundary conditions (3)–(4) have significant specifics. It lies in the fact that they are unseparated (nonlocal) boundary conditions, and this significantly complicates the numerical solution of the problem [7].

Let us assume that the volumes $q^{ks}(t)$, leak locations ξ^{ks} and the corresponding segments of the set $K^{loss} \subset K$ are not known and they need to be determined using additional information about the flow modes given below (we denote the set of such vertices by $I_q^f \subset I^f$):

$$\begin{cases} Q_{mes}^m(t) = Q^{ms}(0,t), & s \in I_m^+, \; if \;\; I_m^- = \varnothing, \\ Q_{mes}^m(t) = Q^{sm}(l^{sm}, t), & s \in I_m^-, \; if \;\; I_m^+ = \varnothing, \end{cases} \qquad m \in I_q^f. \tag{6}$$

Based on the meaning of the problem, we will assume that there are restrictions on the identified functions and parameters:

$$0 \le \xi^{ks} \le l^{ks}, \quad \underline{q} \le q^{ks}(t) \le \overline{q}, \quad t \in [t_0, T], \; (k,s) \in K^{loss}, \tag{7}$$

where $\underline{q}, \overline{q}$ –are the given values.

2. SOLUTION APPROACH

To determine the unknown numbers of areas where leaks occurred, it is necessary to solve the above problem for various options for groups of areas suspected of having leaks. It is clear that the variant in which the smallest value of the functional (1) is obtained corresponds to the solution of the problem posed. In the case of a large number of leaks or a very large number of segments M, such an enumeration on networks of a complex structure is a laborious task. In practice, as a rule, segments suspected of having a possible leak are assigned with sufficient plausibility by experts (specialists) based on experience, information on the duration and modes of operation of the segments, and other considerations.

It is clear that if, as a result of solving problem (1)–(7), with a sufficiently small value of the functional, it will be obtained that $\left|q^{ks}(t)\right| \le \varepsilon$, $t \in [\tau, T]$, ε is a sufficiently small number, then this means that there is no leakage of raw materials in that segment (k,s), as well as in the pipeline as a whole. As a rule, the consequence of the absence of a leak on the (k,s)–th segment is a large minimum value of the functional or, if the obtained value ξ^{ks} is such that $0 \le \xi^{ks} \le \varepsilon$ or $l^{ks} - \varepsilon \le \xi^{ks} \le l^{ks}$.

CONCLUSIONS

A numerical approach to determining the unknown numbers of segments where leaks occurred, leak locations and amounts in an unsteady f luid f low in a pipeline network of complex structure was proposed. The mathematical formulation of the considered problem was reduced to the class of parametric optimal control problems for a system of a large number of hyperbolic partial differential equations. A feature that causes special difficulties in the numerical solution is that the problem involves nonseparated boundary conditions at network nodes, which appear due to an analogue of Kirchhoff's law concerning material balance conservation and the continuity of the pressure value. Another specific feature is that no initial conditions are specified, since the process is too long; additionally, impulse functions are involved in the differential equations. Formulas for the gradient of the cost functional with respect to the identification parameters were obtained, which make it possible to use efficient numerical optimization methods of the first order.

References

1. Aida-zade K.R. Ashrafova Y.R. Numerical Leak Detection in a Pipeline Network of Complex Structure with Unsteady Flow. *Computational Mathematics and Mathematical Physics*, 2017, Vol. 57, No. 12, pp. 1919–1934.
2. Aida-zade K.R. Ashrafova Y.R. Numerical solution to an inverse problem on a determination of places and capacities of sources in the hyperbolic systems. *Journal of industrial and management optimization American Institute mathematical sciences-AIMS,* 2020, V.16, No.6, p.3011-3033.
3. Wichowski R. Hydraulic Transients Analysis in Pipe Networks by the Method of Characteristics (MOC). *Archives of Hydro-Engineering and Environmental Mechanics*, 2006, Vol. 53, No. 3, pp. 267–291.
4. Oyedeko K.F.K. Balogun H.A Modeling and Simulation of a Leak Detection for Oil and Gas Pipelines via Transient Model: A Case Study of the Niger Delta. *Journal of Energy Technologies and Policy*, 2015, Vol.5, No.1.
5. Ashrafova Y.R. Numerical investigation of the duration of the effect exerted by initial regimes on the process of liquid motion in a pipeline. *J. of Eng. Physics and Therm.* 2015. V. 88. No.5, P.1–9.
6. Chaudhry H.M. Applied Hydraulic Transients. *Van Nostrand Reinhold*, New York, 1988.
7. Aida-zade K.R. Ashrafova Y.R. Solving systems of differential equations of block structure with nonseparated boundary conditions. *J.of Applied and Industrial Mathem.* 2015. V. 9. No 1. pp.1–10.