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Prediction and perfection of tribological properties of heavily loaded interacting surfaces

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Abstract. Interacting surfaces of the rubbing elements of various machines often work in the extreme conditions that has a great influence on the machine service life, emergency situations (in transport machines on the traffic safety), energy consumed, preventive and maintenance expenses, pollution of the environment by vibrations, noise and lubricants. Usually, at estimation of tribological properties of the interacting surfaces they are supposed to be known and constant parameters, but as the researches show, the range of their variation is large and difficult-to-predict. Our researches have shown especially high sensitiveness of the surface operational properties towards destruction of the third body existing in the contact zone, which is greatly influenced by its tribological properties and power and thermal loads of the contact zone. In the conditions of destruction of the third body developed by us are expressed kinematic, geometric, power and thermos-physical characteristics of the interacting surfaces and properties of the lubricant. Properties and stability of the boundary layers are expressed by the coefficients ascertained on the base of results of the experimental researches. Were also developed and tested in the laboratory conditions new, ecological, frictional and anti-frictional friction modifiers for steering and tread surfaces of the train wheels and rails.

Keywords**:** *interacting surfaces; third body; contact zone; friction modifier*

1. INTRODUCTION

The working resource of heavily loaded interacting elements of machines depends mainly on tribological properties of their rubbing surfaces. Such elements are gear drive and friction gear, the train wheel and rail, cam mechanisms etc., that fail basically due to the wear of their rubbing surfaces. For providing working capacity of these elements in various working conditions, they select corresponding materials and methods of their mechanical and thermos-chemical treatment, lubricants and methods of lubrication etc., though, obtaining the desirable results is still problematic.

For ascertainment of working capacity of such elements, they mainly use model of Hertz (1892) and conduct calculation on the contact strength. According to the method of calculation, if stress acting in the contact zone of immobile, absolutely smooth, dry and homogeneous bodies exceeds the allowable one, superficial layers of these elements will fail due to fatigue. The formula for calculation of the contact strength contains normal load, reduced module of elasticity of the materials and reduced radius of curvature (Fig. 1a).

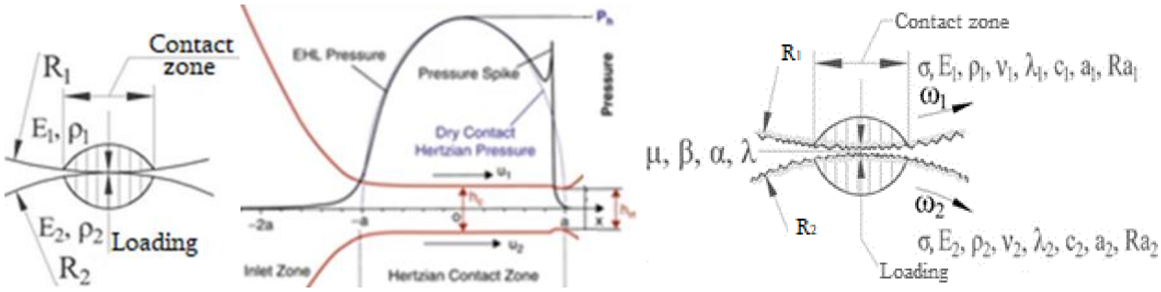

Fig. 1. The contact zone scheme of convex surfaces corresponding to model of Hertz, pressure distribution in EHD and the kinematic, geometric, mechanical and thermo-physical parameters of heavily loaded frictional contact.

Model of Hertz ensures calculation of contact stresses in the conditions of the noted admissions. However, it is clear that such method is inadequate to estimation of working capacity of the interacting surfaces, especially for up-to-date high-speed machines in the conditions of use of high-quality lubricants. The mechanical, thermo-physical and tribo-chemical processes of various generation and permanent formation and destruction of the third body take place in the contact zone in the conditions of high power and thermal loads that have essential influence on tribological properties of these surfaces. However, despite many experimental and theoretical works [1 – 6], devoted to ascertainment of tribological properties of interacting surfaces, these processes are still not properly studied. It should be noted that the results of many experimental researches do not match [7] and the reasons causing this, are vague. Besides, the methods of prediction of tribological properties are characterized by low effectiveness and do not meet requirements in engineering practice due to limitation of acting parameters.

The popular method of estimation of the interacting surfaces working capacity is calculation of surfaces on "contact strength" on the base of Hertz model ensuring possibility of estimation of vertical elastic deformation, contact size, maximum normal and shearing stresses and their distribution in the contact zone for initial linear and point contact (Table 1, [1]). However, this calculating model is valid for absolutely smooth, homogeneous and elastic bodies and at its application for estimation of the interacting surfaces working capacity, the results are obtained with low precision.

Action of the tangential force on the normally loaded contact causes a slip. Before developing a macro-slip, only certain parts of the contact zone perform a local, micro-slip that begins in the places of low-pressure distribution, the rest of the places staying immobile.

In 80-ies of the past century, the researches into similar samples were carried out in 26 laboratories of seven countries. The results of these researches are shown in Fig. 1 [3, 4], from where large deviations of the friction coefficient are seen that, in our opinion, are caused by different rigidities of the testing equipment.

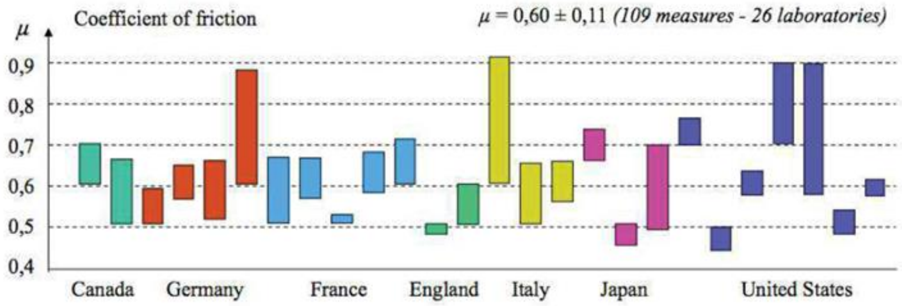

Fig. 2. Comparison of the friction coefficients for the steel rubbing samples

Reliable methods for estimation of the friction and wear rate of the interacting bodies, developed on the base of fundamental laws, do not exist at present and the existent ones are characterized by low informativeness and precision. Lack of the methods for the friction and wear prediction and urgency of solution of the problems stipulate generation of various pragmatic, intuitive and heuristic ideas (for example, the harder material, the lesser wear; the smoother material slides better; high friction means high wear rate etc.) and experimental methods.

2. TRIBOLOGICAL ASPECTS OF WORKING CAPACITY OF MACHINES HEAVILY LOADED INTERACTING SURFACES

In general, tribological properties of the contact zone (friction force, friction coefficient, wear rate etc.) depend on many factors, out of which not all are known. The parameters influencing tribological properties of the contact zone and their impact on the operational properties of interacting elements are shown in the diagram (Fig.2).

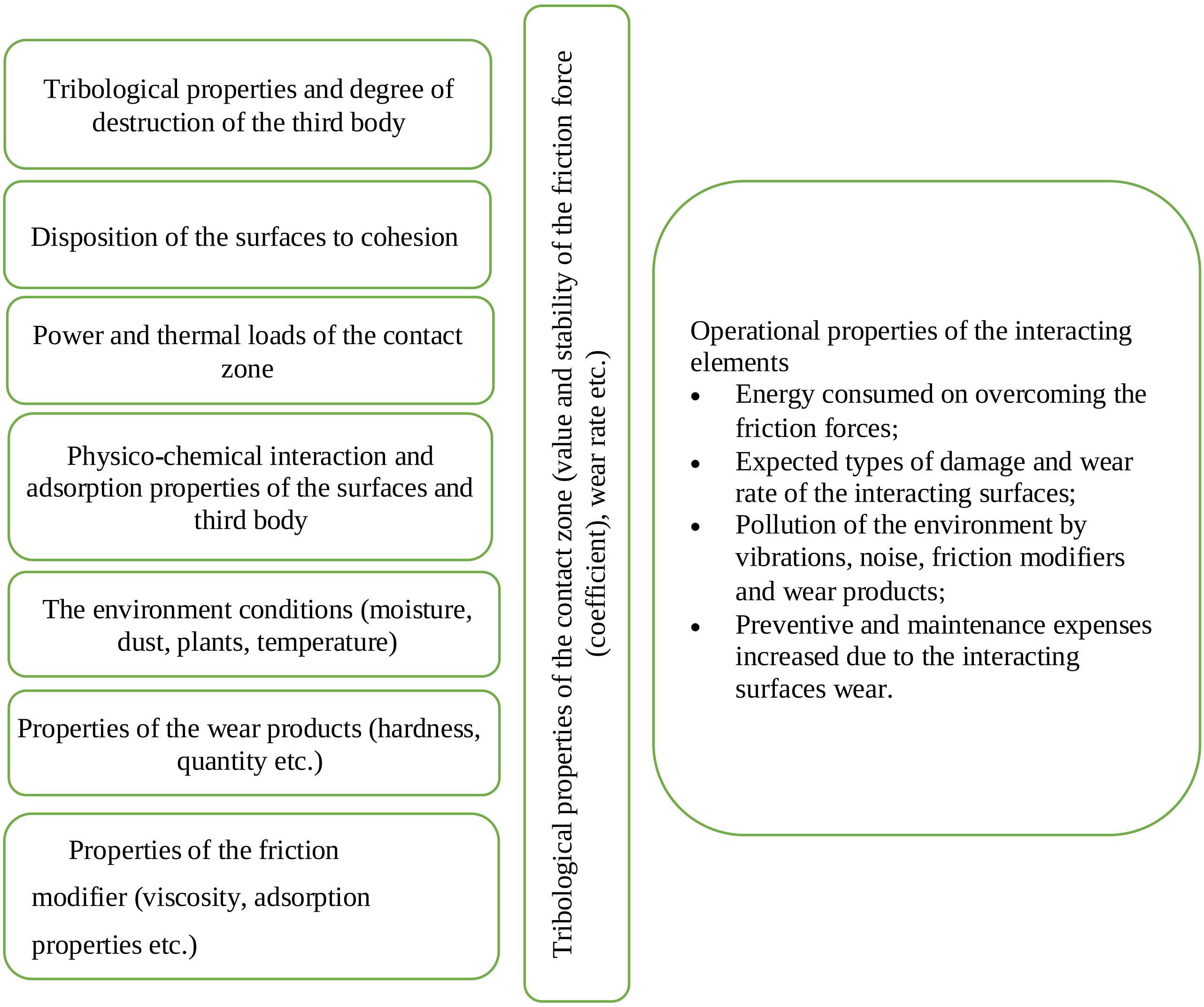

Fig. 3. The parameters acting on the contact zone tribological properties and their influence on operational properties of the interacting surfaces

Besides, operational safety of machines, efficiency, preventive and maintenance expenses, ecological compatibility etc., greatly depend on tribological properties of heavily loaded interacting surfaces of machines. Therefore, prediction and control of tribological properties of such interacting surfaces is topical.

Displacement of the coupled places of surfaces relative to each other causes sharp increase of the shear stresses and corresponding deformations, value and instability of the friction forces and rupture of the coupled places. It is possible in this case transfer of the pulled out material from on surface on the other, sharp change of roughness of these surfaces and development of the process of catastrophic wear – scuffing. The shear deformation generated on the surface sharply decreases towards the depth and multiple repetition of such processes results in superficial plastic deformations, lamination and fatigue damage (Figure 2) [30, 31 In such conditions the scales of destruction and the dominant kind of damage depends on the working conditions.

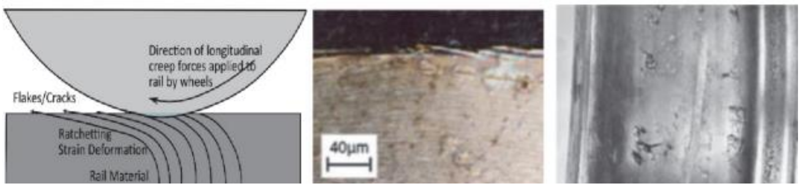

Fig. 4. The scheme of the surface plastic deformation (a); appearance of cracks and lamination (b); appearance of fatigue pits (c).

Thus, roughness of the surfaces of heavy loaded friction contact differs from their initial roughness and depends on the working mode. For provision of the due tribological properties of interacting surfaces, separation them from each other by the continuous third body with appropriate tribological properties is necessary.

The friction forces between interacting surfaces (at lack of the third body in the places of actual contact) depend on the total area of the actual contacts $Ff = \psi$ 05 ($P\tau_{Aasp}$) [24]. Where τ is effective strength on shear of the actual contact area of interacting surfaces; A_{asp} – seizure area of the actual contact that depends on the thermal load of the contact zone, thickness of the heated up layer, properties of the surfaces and environment of individual micro-asperities etc.

Hence, the friction forces depend on the contact area in both cases, when the third body separates the surfaces from each other fully or partially.

It should be noted that various types of surface take place simultaneously and proceed with various intensity and a dominant type of damage ascertained visually. The experimental researches have shown that damage intensity and type, of interacting surfaces are especially sensitive to the relative sliding velocity and shear stresses. Thereat, at low total and relative sliding velocities of the surfaces, when power of the thermal action, velocity and resistance of the shear deformation in the contact zone are comparatively small, stability of the third body and its resistance to scuffing are high and a main type of damage is fatigue wear [4]. With increase of the total and relative sliding velocities of surfaces, thermal load of the actual contact zone and destruction intensity of the third body increases. However, time of action of this load, thickness of the heated up layer and sizes of micro-asperities generated because of the scuffing and subsequent rupture of the seized places, decrease. Such phenomena take place on tread surfaces of the train wheel, near the pitch point of the gear drives, in the rolling bearings etc. (Figure 3). At increase of the relative sliding velocity, share of the adhesive wear and scuffing increases and it often becomes a dominant type of damage. For example, a steering surface of the train wheel, places of tooth profile of the gear drive distant from the pitch point, cam mechanisms etc.

Fig. 5. The damage types: (a) train wheel with fatigue damage of the tread surface and adhesive wear (scuffing) of the flange; (b) gear wheel with the traces of scuffing on the tooth face; (c) inner ring of the rolling bearing with the traces of fatigue damage.

For avoiding the above-mentioned non-desirable phenomena, providing the contact zone with the third body having due properties, its protection against destruction and control of the friction coefficient are necessary. However, despite the great number of scientific works this direction could not attract due attention of the scientists until today. Variation of tribological properties of the surfaces is a result of various mechanical, physical and chemical processes proceeding simultaneously in the contact zone, whose essence and mechanism of action are not properly studied [20–22]. This complicates control of the mentioned processes that needs consideration of many factors acting simultaneously. Such factors are:

- Initial tribological properties of the third body and surfaces; influence of interaction of the friction modifier and other materials existent in the contact zone and the surfaces on the properties and stability of the third body and surfaces.
- Structural, physical and mechanical peculiarities and tendency to scuffing of the clean (juvenile) surfaces in the places of the third body destruction; destruction peculiarities of the seized places;
- Influence of the contact zone working conditions on the wear type and rate, variation of the micro- and macro-geometry etc.

3. ESTIMATION OF STABILITY OF THE THIRD BODY ON THE BASE OF EHD THEORY OF LUBRICATION

The most interesting practical aspect of the EHD lubrication theory is the determination of lubricant film thickness which separates the bodies. Calculation of the oil film thickness is the main problem of the EHD lubrication theory and there are a numerous literature sources about it. In EHD lubrication, the load is carried by the elastic deformation together with the hydrodynamic action of the lubricant (Dowson, 1995; Hamrock and Dowson, 1981). There are various formulas for isothermic and anisothermic solutions for EHD problems describing the behavior of oil film thickness with various accuracy. The formulas for calculation of oil film thickness obtained by different authors and the results obtained by their use, and the results of experimental research of oil film thickness are given in the table. There are number of formulas that meet isothermal or an-isothermal tasks of EHD theory of lubrication, which describe the lubricant film thickness with various accuracy. In the Table 3. You can see the oil film thicknesses (μm) calculated using the formulas by various authors and the relevant results of experimental researches in the conditions of frictional rolling.

The formulas of the first four authors given in the Table do not take into account thermal phenomena proceeding in the contact zone and they give the results with the same precision on the low rolling velocities. At high rolling velocities (10 - 20 m/s) the oil viscosity, velocity and other parameters stipulate the thermal processes in the oil layer that first decreases intensity of the oil layer growth and then decreases it. Operation of the interacting bodies in the conditions of rolling-sliding activates significantly the thermal phenomena and isothermal solution of the EHD theory equations is unacceptable in this case.

Table 2. Oil film thicknesses

Author	Formula	Sum rolling speed m/c						
			5	10	20	30	50	70
Ertel and Grubin [65]	$\frac{h}{R}=1{,}19\left[\frac{\mu_0 V_\Sigma \beta}{P}\right]^{8/11}\left[\frac{ER}{P}\right]^{1/11}$	0,44	1,41	2,34	3,89	5,23	7,6	9,7

Petrusevich [66]	$h=\frac{\left(\frac{\nu}{100}\frac{V_{\Sigma}}{1000}\right)^{2/3}}{\left(\frac{a_{bx}}{250}\right)^{0,5}}\;\frac{\left(\frac{R}{5}\right)}{\left(\frac{\sigma_g}{10^4}\right)^{0,4h}}{}^{0,25}$	0,34	0,98	1,56	2,47	3,24	4,556	5,7
Dayson and Higginson [67]	$\frac{h}{R}=1,6\frac{(\beta E)^{0,6}(\mu V_{\Sigma}/ER)^{0,7}}{(P/ER)^{0,13}}$	0,55	1,7	2,74	4,45	5,9	8,45	10,7
Kodnir [68]	$h=2,224\frac{(\mu_0 V_{\Sigma})^{0,75}\beta^{0,6}R^{0,4}}{P^{0,15}}$	0,24	0,82	1,385	2,33	3,16	4,3	6,0
Murch and Wilson [69]	$\frac{h}{R}=1,19\left(\frac{\mu_0 V_{\Sigma}}{P}\right)^{8/11}\left(\frac{ER}{P}\right)^{1/11}\left(\frac{3,94}{3,94+V^{0,68}}\right)$	0,44	1,36	2,146	3,2	3,87	4,578	4,85
Drozdov, Tumnishvili [70]	$\frac{h}{R}=1,57\left(\frac{\mu_0 V_{\Sigma}}{P}\right)^{0,7}\left(\frac{P\beta}{R}\right)^{0,6}\left(\frac{\lambda}{\alpha\mu_0 V^2}\right)^{0,36}$	0,5	1,4	2,4	2,36	2,32	2,25	2,2
Experiments of Tumanishvili		0,6	1,7	2,6	2,7	2,65	2,6	2,6

Until 70-s of the last century the oil layer of hydrodynamic generation existent in the contact zone, was considered as a parameter determining a working capacity of the heavy loaded frictional contact. Many experimental and theoretical works are devoted to study of thickness of this layer [1-5]. An approximate (digital) solution of the elasto-hydrodynamic problem considering thermal processes is given in the works [6 11], where the temperature, pressure and thickness of the oil layer between the cylinders interacting with the rolling-sliding friction, are determined. However, in spite of many attempts, ascertainment of the reliable relations between the thickness of the oil layer and tribological properties of the contact zone turned out to be problematic [7 12].

The supplements to the lubricants developed in succeeding years and technical means of studying the processes proceeding in the contact zone have radically widened the scope of the researches. At common operational conditions, various types of boundary films - products of interaction with the environment that prevent the direct contact of rubbing surfaces, cover these surfaces with thin layers. Depending on the friction conditions, properties of the surfaces and environment, these layers may have various tribological properties that will have the great influence on the boundary friction [8- 10]. This is confirmed by the results of the experimental researches in the inert gas environment and vacuum, that excludes the possibility of oxidation during friction. Under such conditions, the seizure and intensive wear rate are observed. To prevent these undesirable phenomena, it is necessary to provide the presence of the third body in the contact zone with due properties, control of the friction factor and protection of the third body from destruction.

The lubrication allows to prevent wear, premature fatigue and reduces power loss by forming the lubrication film on the surface avoiding direct contact of surfaces. Until recently, it was widely believed that a lubricating film of hydrodynamic or elastic-hydrodynamic (EHD) origin protects the interacting surfaces from direct contact and improves their tribological properties if its thickness exceeds the height of wet roughness.

The first approximate solution of the integro-differential equations describing the EHD processes of lubrication (considering the thermal processes in the contact zone and dependence of the oil viscosity on the pressure and temperature) belongs to Chang [11]. However, such approach does not take into account existence and stability of the boundary layers in the contact zone and the obtained results are not precise [12].

With the advent of additives that significantly improve the properties of surfaces, the importance of boundary layers was recognized, and then a new term appeared - the "third body", which unites all materials located in the contact zone. It particularly concerns to the adhesive wear, which is emerged at destruction of the third body and direct contact of the juvenile surfaces. The third body destruction criterion, which is developed on the base of elasto-hydrodynamic theory of lubrication and results of experimental researches considering stability of the boundary layers, has the form:

$$K \cdot V_{\Sigma k}^{a} \cdot V_{sl}^{b} \cdot P_{ll}^{c} \cdot \mu 0\, d \cdot Rl \cdot (\sqrt{R_{a1}{}^{2}+R_{a2}{}^{2}})\, f \cdot \beta g \cdot \lambda h \cdot \alpha i \cdot aj \cdot En \le 1 \qquad (1)$$

where $V_{\Sigma k}$ is total rolling velocity; V_{sl} – sliding velocity; P_{ll} – linear load; μ - dynamic viscosity of the lubricant; R – reduced radius of curvature of the surfaces; R_{a1} and R_{a2} - average standard deviation of the interacting surfaces; β – piezo coefficient of the lubricant viscosity; λ – the lubricant thermal conductivity; α – thermal coefficient of the lubricant viscosity; a – thermal diffusivity. The degrees a, b, c, . . . n and coefficient K are specified on the base of the experimental data obtained by T. I. Fowle, Y. N. Drozdov, Vellawer, G. Niemann, El-Sisi, A. I. Petrusevich, I. I. Sokolov, K. Shawerhammer, G. Tumanishvili and given in the Table 1.

Table 3. Values of degrees of the parameters

a	b	c	D	e		g	h	i	j	n
0.37- 0.61	(-0.36)-(-1.32)	(-0.18)-(0.66)	0.04-0.52	0.23-0.36	1	0.6	0.18-0.66	(-0.18)-0.66)	0.09-0.33	0.045-0.165

As it is seen from the Table 1 that destruction of the third body is especially sensitive to the sliding velocity. The rolling velocity, linear loading, roughness, viscosity, thermo-physical characteristics of the contacting surfaces and third body have relatively less influence and as for the reduced radius of curvature of the discs and modulus of elasticity, they have even less influence on the destruction of the third body. The shear deformation generated on the surface sharply decreases towards the depth and multiple repetition of such processes results in superficial plastic deformations, lamination and fatigue damage (Figure 4) [12, 13 F. Braghin a, R. Lewis b, R.S. Dwyer-Joyce b, S. Bruni. A mathematical model to predict railway wheel profile evolution due to wear. Wear 261 (2006) 1253–1264. Lewis R., Dwyer-Joyce R.S., Bruni S., Ekberg A., Cavalletti M., Bel Knani K.. A New CAE Procedure for Railway

Wheel Tribological Design. 14th International Wheelset Congress, 17-21 October, Orlando, USA.]. Usually, it is a general assumption to consider the coefficient of friction as a known and constant value. This hypothesis is clearly not correct as many factors can change the friction coefficient. The various dominant damage types, wear rates, and friction coefficients are typical for various relative sliding. In Figure 6 is shown dependence of the friction coefficient on the relative sliding and expected kind of surface damage. Three zones can be distinguished in Figure 4. In zone 1 and at the beginning of zone 2, deformations of the subsurface layers reach the maximum values, and the interacting surfaces undergo cyclic deformations. With the rise of relative sliding, the contact temperature gradually increases, decreasing viscosity of the third body [14 Eadie DT, Kalousek J, Chiddik KC.The role of high positive friction (HPF) modifier in the control of short pitch corrugations and related phenomena. Wear. 2002; 253:185-192.] and the friction factor that reaches the minimum value. At full separation of the interacting surfaces by the third body, the tribological properties of the contact zone mainly depend on the properties of the third body, and they provide high wear resistance of the interacting surfaces and relatively stable friction coefficient.

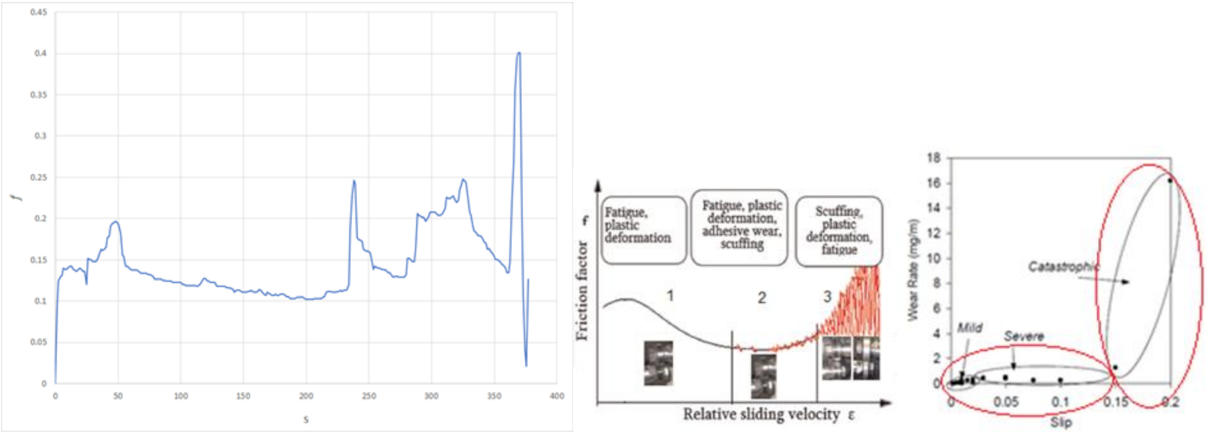

Fig. 6. Dependences of the friction coefficient (f) on the sliding distance at a load of 40 N, an initial temperature of 130^0C for a friction modifier based on polyethylene glycol of our manufacture (a), a generalized dependence of the friction coefficient (f) on the relative slip (ε) and the expected types of surface damage and slip wear rate (in percent) (c).

In zone 2 the separate small impulses of the friction moment and adhesive wear of low intensity correspond to destruction of the third body in the separate unique and multiple places. The first point marks of scuffing appear on the interacting surfaces and balance between destruction and restoration of the third body is observed, that stipulates the “mild” and “sever” wear [15 Lewis R, Dwyer-Joyce RS. Wearmechanisms and transitions in railwaywheel steels. Proceedings of theinstitution of mechanical engineers, partJ. Journal of Engineering Tribology.2004; 218(6):467-478.]. In zone 3 destruction of the third body takes place in the narrow strips that pass then into whole area of interacting surfaces, the surface roughness changes correspondingly to working conditions (rolling speed, sliding rate, contact pressure etc.) resulting in rise of thefriction coefficient, its instability, wear rate (reaching “catastrophic” wear), and scuffing.

Therefore, we have three stages of variation of the tribological characteristics of interacting surfaces (friction coefficient, roughness and wear): at continuous third body, at reversible discontinuous third body and at irreversible discontinuous third body. The first stage is characterized by minimal wear rate and change of roughness and stable friction factor. The second stage is characterized also by the minimal but slightly unstable friction coefficient, roughness, and wear rate. In terms of tribological characteristics, the stage 1, as well as stage 2 can be considered as the acceptable working conditions of the tribological system. In contrast to this, stage 3 is characterized by the sharp rise of the constant and variable components of the friction coefficient, wear rate (“catastrophic wear”), vibrations, and noise and operation in this zone is not admissible.

The signs of beginning of the third body destruction are instability of the friction (coefficient) moment, vibrations, and noise, and at visual observation in the laboratory conditions, the signs of scuffing are noticeable. Its prediction is possible with the use of the tables and graphs considering the given friction modifier, working conditions and environment properties, as well as the criterion of destruction of the third body [16 Tumanishvili G, Natriashvili T, Nadiradze T. Perfection of technical characteristics of the railway transport system Europe-Caucasus-Asia (TRACECA) in the book. In: Sladkowski A, editor. Transport Systems and Delivery of Cargo on East-West Routes, Springer. 2018. pp. 303-368].

4. CONCLUSION

1. At present does not exist reliable methods of estimation friction and wear rate of interacting surfaces, based on the fundamental laws and existent methods are characterised by low informativenes and precision.

2. For provision of due tribological properties of interacting surfaces, their separation from each other by the continuous third body with appropriate tribological properties is necessary.
3. Three stages of change in the tribological characteristics of interacting surfaces are established depending on degree of destruction of the third body, determined experimentally in the laboratory conditions. The first and second stages of destruction of the third body provide acceptable tribological characteristics of the interacting surfaces, and the third stage of destruction of the third body corresponds to catastrophic wear and is unacceptable.

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