The Dynamic Model of GDP for Azerbaijan

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***Abstract*—One of the indicators of the country's economic growth is the gross domestic product (GDP), and one of the factors of economic growth is capital. The main criteria and source of economic development is economic growth. Economic growth is a sustainable increasing tendency of the main indicators of national theory production (GDP, GNI). Furthermore, absolute value and growth per capita are also considered. In economic and statistics, various indicators are used to measure the volume of national production. The most important of these is gross domestic product (GDP). GDP is expressed by monetary unit of the final products and services produced in the economy. Here should be taken into account the fact that GDP comprises final products and services produced within the particular country. In this work, was created the dynamic model that demonstrates the dependence of GDP on investments in case of Azerbaijan economy.**

Keywords—gross domestic product, macroeconomics, dynamic model, optimal trajectory, mathematical modeling of economic systems

I. Introduction

Consider the following task: how much investment should be allocated in a certain year to achieve the desired level of GDP after a certain period of time?

To do this, consider the following task:

$$J=\sum_{i=0}^{N}\left(x_{jel,i}-x_i\right)^2+\sum_{i=0}^{N-1}u_{i+1}^2\to\min \tag{1}$$

$$x_{i+1}=Fx_i+Gu_{i+1}+v,\ i=\overline{0,n-1} \tag{2}$$

$$x(0)=x_0 \tag{3}$$

Here, u_i is the volume of investment and x_i is the volume of GDP in the i-th year.

II. THE MODEL OF THE DESIRED LEVEL OF GDP

Since we want to achieve the desired level at the end of the trajectory ($x_{jel,i}=0, i=\overline{1,N-1}, x_{jel,N}=x_{jel}$), we can write (22) as follows:

$$J=\left(x_{jel}-x_N\right)^2+\sum_{i=0}^{N-1}\left(x_i^2+u_{i+1}^2\right)\to\min \tag{4}$$

We can rewrite this problem in the following form:

$$J=\frac{1}{2}q\left(x_{jel}-x_N\right)^2+\sum_{i=0}^{N-1}\left(k_1x_i^2+k_2u_{i+1}^2\right)\to\min \tag{5}$$

$$x_{i+1}=Fx_i+Gu_{i+1}+v,\ i=\overline{0,n-1} \tag{6}$$

$$x(0)=x_0 \tag{7}$$

Here, q, k_1, k_2 are coefficients, F, G, v was defined in [10], x_{jel} is the desired level of GDP and N is the number of years. For this, we construct an extended criterion of quality $\bar{J}$ [8]. To do this, we add systems of equations with coefficients $\lambda(i)$[1, 7] to function J:

$$\bar{J}=\frac{1}{2}q\left(x_{jel}-x_N\right)^2+\sum_{i=0}^{N-1}\left[\frac{1}{2}\left(k_1x_i^2+k_2u_{i+1}^2\right)+\lambda_{i+1}\left(Fx_i+Gu_{i+1}+v-x_{i+1}\right)\right] \tag{8}$$

We use the following notation:

$$\Phi\left(x(N)\right)=\frac{1}{2}q\left(x_{jel}-x_N\right)^2$$

$$H^i=\frac{1}{2}k_1x_i^2+\frac{1}{2}k_2u_{i+1}^2+\lambda_{i+1}\left(Fx_i+Gu_{i+1}+v\right)$$

We can rewrite (8) as such:

$$\bar{J}=\frac{1}{2}q\left(x_{jel}-x_N\right)^2-\lambda_Nx_N+\sum_{i=1}^{N-1}\left[\frac{1}{2}\left(k_1x_i^2+k_2u_{i+1}^2\right)+\lambda_{i+1}\left(Fx_i+Gu_{i+1}+v\right)-\lambda_ix_i\right]+H^0\to\min \tag{9}$$

We get the following problem:

$$\bar{J}=\frac{1}{2}q\left(x_{jel}-x_N\right)^2-\lambda_Nx_N+\sum_{i=1}^{N-1}\left[\frac{1}{2}\left(k_1x_i^2+k_2u_{i+1}^2\right)+\lambda_{i+1}\left(Fx_i+Gu_{i+1}+v\right)-\lambda_ix_i\right]+H^0\to\min \tag{10}$$

$$x_{i+1}=Fx_i+Gu_{i+1}+v,\ i=\overline{0,n-1} \tag{11}$$

$$x(0)=x_0 \tag{12}$$

To solve the problems (10)-(12), i.e. to find the values $\lambda_i,(\ i=\overline{0,n+1})$, $u_i,(\ i=\overline{0,n-1})$ and $x_i,(\ i=\overline{0,n})$, we need to solve the following system of equations [3]:

$$\frac{\partial H^i}{\partial x_i} = \lambda_i \quad (13)$$

$$\frac{\partial H^i}{\partial u_i} = 0 \quad (14)$$

$$\frac{\partial \phi}{\partial x_N} = \lambda_N \quad (15)$$

From this we get:

$$\lambda_i = k_1 x_i + \lambda_{i+1} F \quad (16)$$

$$0 = k_2 u_{i+1} + \lambda_{i+1} G \quad (17)$$

$$\lambda_N = q(x_N - x_{jel}) \quad (18)$$

And from (17) we find:

$$u_{i+1} = -\lambda_{i+1} G k_2^{-1} \quad (19)$$

Using (19) in (11) we find,

$$x_{i+1} = Fx_i - G^2 k_2^{-1} \lambda_{i+1} + v, i = \overline{0, n-1}$$

In (16) we do the following conversion:

$$F\lambda_{i+1} = -k_1 x_i + \lambda_i$$

Based on these transformations, we obtain the following system of equations:

$$\begin{cases} x_{i+1} = Fx_i - G^2 k_2^{-1} \lambda_{i+1} + v, i = \overline{0, n-1} & (20) \\ F\lambda_{i+1} = -k_1 x_i + \lambda_i & (21) \end{cases}$$

Here we find:

$$\begin{cases} x_{i+1} + G^2 k_2^{-1} \lambda_{i+1} = Fx_i + v, i = \overline{0, n-1} \\ F\lambda_{i+1} = -k_1 x_i + \lambda_i \end{cases}$$

We write the final system in form of matrices:

$$\begin{bmatrix} E & G^2 k_2^{-1} \\ 0 & F \end{bmatrix} \begin{bmatrix} x_{i+1} \\ \lambda_{i+1} \end{bmatrix} = \begin{bmatrix} F & 0 \\ -k_1 & E \end{bmatrix} \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix}.$$

From this we get x_{i+1} and λ_{i+1}:

$$\begin{bmatrix} x_{i+1} \\ \lambda_{i+1} \end{bmatrix} = \begin{bmatrix} E & G^2 k_2^{-1} \\ 0 & F \end{bmatrix}^{-1} \begin{bmatrix} F & 0 \\ -k_1 & E \end{bmatrix} \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + \begin{bmatrix} E & G^2 k_2^{-1} \\ 0 & F \end{bmatrix}^{-1} \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (22)$$

From (22) we get:

$$\begin{bmatrix} x_{i+1} \\ \lambda_{i+1} \end{bmatrix} = \begin{bmatrix} F + G^2 k_1 k_2^{-1} F^{-1} & -G^2 k_2^{-1} F^{-1} \\ -k_1 F^{-1} & F^{-1} \end{bmatrix} \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (23)$$

We introduce the following notation:

$$A = \begin{bmatrix} F + G^2 k_1 k_2^{-1} F^{-1} & -G^2 k_2^{-1} F^{-1} \\ -k_1 F^{-1} & F^{-1} \end{bmatrix}$$

Then, (23) can be written as such:

$$\begin{bmatrix} x_{i+1} \\ \lambda_{i+1} \end{bmatrix} = A \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (24)$$

We find:

$$\begin{bmatrix} x_{i+2} \\ \lambda_{i+2} \end{bmatrix} = A \begin{bmatrix} x_{i+1} \\ \lambda_{i+1} \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} = A^2 \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + A \begin{bmatrix} v \\ 0 \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} x_{i+3} \\ \lambda_{i+3} \end{bmatrix} = A \begin{bmatrix} x_{i+2} \\ \lambda_{i+2} \end{bmatrix} + \begin{bmatrix} v \\ 0 \end{bmatrix} = A^3 \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + (A^2 + A + A^0) \begin{bmatrix} v \\ 0 \end{bmatrix}$$

From here we can simply write:

$$\begin{bmatrix} x_{i+k} \\ \lambda_{i+k} \end{bmatrix} = A^k \begin{bmatrix} x_i \\ \lambda_i \end{bmatrix} + (A^{k-1} + A^{k-2} + \dots + A + A^0) \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (25)$$

And so, (25) can be written in the following form:

$$\begin{bmatrix} x_N \\ \lambda_N \end{bmatrix} = A^N \begin{bmatrix} x_0 \\ \lambda_0 \end{bmatrix} + (A^{N-1} + A^{N-2} + \dots + A + A^0) \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (26)$$

We introduce the next notation:

$$A^N = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix},$$

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = (A^{N-1} + A^{N-2} + \dots + A + A^0) \begin{bmatrix} v \\ 0 \end{bmatrix} \quad (27)$$

So, (26) can be written as:

$$\begin{bmatrix} x_N \\ \lambda_N \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_0 \\ \lambda_0 \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \quad (28)$$

From this we get:

$$\begin{cases} x_N = a_{11} x_0 + a_{12} \lambda_0 + f_1 \\ \lambda_N = a_{21} x_0 + a_{22} \lambda_0 + f_2 \end{cases}$$

If we add condition (18) here, we obtain the following system of equations:

$$\begin{cases} x_N = a_{11} x_0 + a_{12} \lambda_0 + f_1 & (29) \\ \lambda_N = a_{21} x_0 + a_{22} \lambda_0 + f_2 & (30) \\ \lambda_N = q(x_N - x_{jel}) & (31) \end{cases}$$

The values of f_1 and f_2 are obtained from (27). Considering (29) and (30) in (31) we get:

$$\lambda_0 = (qa_{12} - a_{22})^{-1} \left(x_{jel} q - (qa_{11} - a_{21}) - (qf_1 - f_2) \right) \quad (32)$$

x_0 is given to us as an initial condition, and λ_0 can be calculated from (32). Using this, we can calculate $\lambda_i, (i = \overline{0, n+1})$, $u_i, (i = \overline{0, n-1})$ and $x_i, (i = \overline{0, n})$. Values of $\lambda_i, (i = \overline{0, n+1})$ and $x_i, (i = \overline{0, n})$ can be calculated from expression (23), and $u_i, (i = \overline{0, n-1})$ from expression (19).

III. CONCLUSION

This approach provides an opportunity for strategic planning of GDP for the country. In this work, to achieve the desired level of GDP, the volume of investment is used as the independent variable in the dynamic model. But as indicated above, many other factors affect GDP. We chose one of them: the amount of investment. But even so, the dynamic model of the optimal GDP trajectory yielded good results (see Table 2 and Figure 1).

Further research will take into account the other\ most influential factors on GDP. In this case, a dynamic model of the optimal trajectory of GDP will give even more adequate results. Many parameters of the incoming model are approximate. Therefore, in the future, work can be developed with fluctuations in parameters - in other words, the study of stability with respect to the change in error (see [10]). Another direction for research is the application of pattern recognition methods with predetermined threshold numbers. In this case, the classification problem is obtained (see [11]).

REFERENCES

[1] Aliev F.A., Larin V.B.,Optimization of Linear Control Systems: Analytical Methods and Computational Algorithms, Amsterdam: Gordon and Breach Sci, 1998.

[2] Aliyev F.A., Metody resheniya prikladnykh zadach optimizachii dinamicheskikh sistem, Baku, 1989 (Aliev F.A., Solution methods of optimization dynamic systems applied problems, Baku, 1989) (in Russian)

[3] Aliyev F.A., Shafizade E.R. Shikhlinskaya R.Y., Murtuzaliyev T.F., Ekonomiko-matematicheskaya model maksimizachii pribyli Internet magazina, Aktualnye problemy ekonomiki, ISSN 1993 – 6788, 3(141), 2013, str.210-219 (Aliev F.A., Shafizade E.R. Shikhlinskaya R.Y., Murtuzaliev T.F.,Economic-Mathematical model of maximization revenue of e-store, Actual problems of Economics, ISSN 1993 – 6788, 3(141), 2013, pp.210-219)(in Russian)

[4] Aliyev F.A., Niftiyev A.A., Shafizade E.R. Shikhlinskaya R.Y., Murtuzaliyev T.F., Ekonomiko-matematicheskaya model v virtualnom biznese s primeneniyem nechetkogo apparata, Desyataya Mejdunarodnaya Aziatskaya Shkola-Seminar “Problemy optimizachii slojnykh sistem”, Kyrgyziskaya Respublika, 2014, str. 33-39 (Aliev F.A., Niftiev A.A., Shafizade E.R. Shikhlinskaya R.Y., Murtuzaliev T.F., Economic-Mathematical model in virtual business with fuzzy apparatus, 10-th International Asian School-Seminar “Optimization

problems of difficult sistems", Kyrgyz Republic, 2014, pp. 33-39) (in Russian)

[5] Aliyev F.A., Niftiyev A.A., Shikhlinskaya R.Y., Shafizade E.R., Murtuzaliyev T.F., Nechetkaya model elektronnoy torqovli, Proceedings of Inatitute of Applied Mathematics, V3, N1, 2014, pp.23-31 (Aliev F.A., Niftiev A.A., Shafizade E.R. Shikhlinskaya R.Y., Murtuzaliev T.F., Fuzzy model of e-trade, Proceedings of Inatitute of Applied Mathematics, V3, N1, 2014, pp.23-31) (in Russian)

[6] Aliev F.A., Shafizadeh E.R., Hasanova G.R., Zeynalov J., Fuzzy model and algorithm for solving of e-shop income maximization, The 5-th International Conference on Control and Optimization with Industrial Applications, 27-29 August, 2015, Baku, Azerbaijan, pp.179-185.

[7] Brayson A., Kho Yu-Shi, Prikladnaya teoriya optimalnogo upravleniya, Moskva, 1972 (Bryson A., Ho Yu-Chi, Applied Optimal Control, Moscow, 1972) (in Russian)

[8] Golichenko O.G., Ekonomicheskoye razvitiye v usloviyakh nesovershennoy konkurenchii: Podkhody k mnogourovnevomu modelirovaniyu, M:Nauka, 1999 (Golichenko O.G., Economic development under imperfect competition. Approximitation to multilevel modeling, M:Nauka, 1999) (in Russian)

[9] Gurman V.I., Osnovy makroekonomicheskogo analiza, Tver, 1995 (Gurman V.I., Basics of macroeconomic analyze, Tver, 1995)

[10] Shafizade E.R., Hasanova G., Dynamic model for Gross Domestic Product in Azerbaijan, 6th International Conference on Control and Optimization with Industrial Applications, COIA- 2018, 11-13 July, Baku, Azerbaijan (Web of Science)

[11] Zang V.B., Sinergeticheskaya ekonomika. Vremya i peremeny v nelineynoy ekonomicheskoy teorii, M: Mir, 1999 (Zang V.B., Synergetic economics. Time and changes in nonlinear economical theory, M: Mir, 1999) (in Russian)