International Conference on Problems of Logistics, Management and Operation in the East-West Transport Corridor (PLMO), October 27-29, 2021
http://conf.cyber.az

The Perturbed Markov Random Walk Described by the Autoregressive Process AR(1) with Finance Application

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***Abstract*—In this paper, we consider a perturbed Markov random walk described by an autoregressive process AR(1) with finance application. The solution of many management problems is brought to autoregressive process.**

Keywords—autoregressive process, AR(1), Markov walk, finance, R programming language

I. INTRODUCTION

Frequently, the values of a time series are highly correlated with the values that precede and succeed them. This type of correlation is called autocorrelation. Autoregressive modeling is a technique used to forecast time series with autocorrelation. A first-order autocorrelation refers to the association between consecutive values in a time series. A second-order autocorrelation refers to the relationship between values that are two periods apart. A pth-order autocorrelation refers to the correlation between values in a time series that are p periods apart. You can take into account the autocorrelation in data by using autoregressive modeling methods.

II. FORMULATION AND PROOF OF THE MAIN RESULTS

Let $\xi_n, n \geq 1$ be a sequence of independent and identically distributed random variables defined on the some probability space $(\Omega, \mathcal{F}, P)$. It is well known (see, for example, Melfi (1992), Novikov (2008), Zhang and Yang (2010), Saadatmand et al (2016)) that the first-order autoregressive process $(AR(1))$ is defined using a recurrent relation of the form

$$X_n = \beta X_{n-1} + \xi_n, \ \ n \geq 1,$$

where $\beta \in R = (-\infty, +\infty)$ is some fixed number and it is assumed that the initial value X_0 does not depend on the innovation $\{\xi_n\}$.

III. PREPARE YOUR PAPER BEFORE STYLING

Defination **(**Melfi (1992)**).** Let a sequence of random variables $Y_0, Y_1, \ldots$ form a Markov chain in R, a sequence of random variables $Z_1, Z_2, \ldots$ has the property that the conditional distribution of the random variable Z_k for each fixed k, provided that (relatively). $\{Y_i, i \geq 0\}$ and $\{Z_i, i \neq k\}$ depends only on Y_k and Y_{k-1}. In this case the sequence of sums $L_n = Z_1 + \cdots + Z_n, n \geq 1$ is called a Markov random walk.

In particular, from this definition it follows that for any Borel function $f: R \times R \to R$ the sum of random variables $Z_k = f(Y_{k-1}, Y_k)$, $L_n = \sum_{k=1}^{n} Z_k$ is a Markov random walk. Note that the usual random walk described by the sums of independent random variables $\xi_n, n \geq 1$ is a Markov random walk. To make sure of this, it is enough to take $f(x_1, x_2) = x_1 - x_2$ and $Y_k = \sum_{i=1}^{k} \xi_i, (Y_0 = 0)$.

It is known that important problems in the theory of nonlinear renewal for an ordinary random walk were studied in the works of Lai and Siegmund (1977), Lallay (1982), Zhang (1988), Woodroofe (1982), (1990) etc. Internal development of renewal theory and the theory of boundary value problems for random walk, as well as new statistical applications indicate the need to study general concepts (the case of dependent cases of random variables, the case of nonlinear boundaries).

Some aspects of Markov renewal and renewal-reward processes also consider in papers Rahimov et al. (2019), (2020), Aliyev and Khaniyev (2013), Aliyev et al (2015), Aliyev and Bayramov (2017).

Recently, much attention has been paid to the study of nonlinear boundary value problems for Markov random walks described by $AR(1)$ processes (see, for example, Novikov (2008), Rahimov et al. (2015), (2019)). It should be noted that the nonlinear boundary value problems for the Markov random walk have been studied much less than for the ordinary random walk described by sums of independent random variables.

Table 1. Descriptive analyses of S&P and NASDAQ

	Highest:	Lowest:	Difference:	Average:	Change %:
S&P500	4,544	2,191.6	2,352.72	3,047	92.73
NASDAQ	15,288	5,398	9,890.10	8,852	183.5

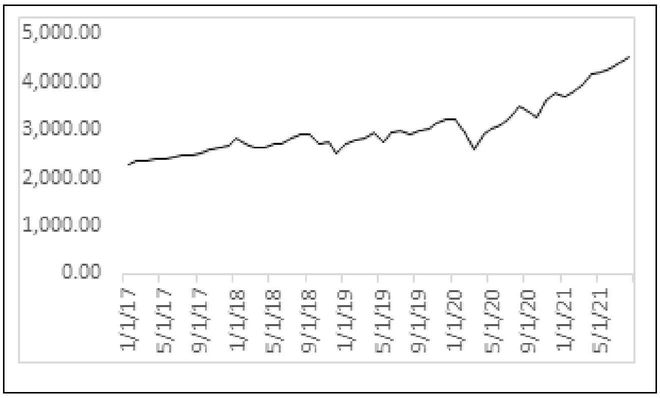

Fig. 1. Price of S&P500 index beetween 01/01/2017-01/08/2021.

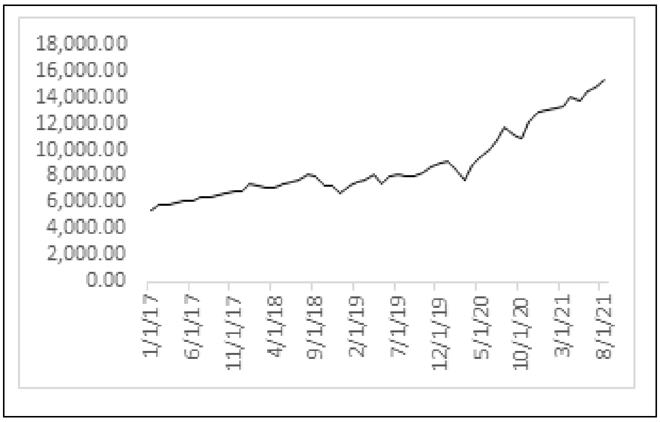

Fig. 2. Price of NASDAQ index beetween 01/01/2017-01/08/2021.

In this paper, we consider a family of moments of the first crossing of a parabolic boundary by a perturbed Markov random walk described by an autoregressive process. The central limit theorem is proved for a perturbed Markov random walk, and the limit behavior of this family of moments of intersection of the parabolic boundary by this walk is studied. The proved Theorem 3 can be used to test the statistical hypothesis for the relationally unknown parameter β of the autoregressive process in a sequential repeated significance test.

ACKNOWLEDGMENT

This work supported by the Science Development Foundation under the President of the Republic of Azerbaijan - **Grant No EIF-ETL-2020-2(36)-16/05/1-M-05.**

REFERENCES

[1] Aliyev, R., Khaniyev, T. (2013) On the Moments of a Semi-Markovian Random Walk with Gaussian Distribution of Summands. Communications in Statistics - Theory and Methods, 43(1): 90–104.

[2] Aliyev, R., Ardic, O., Khaniyev, T. (2015) Asymptotic approach for a renewal-reward process with a general interference of chance. Communications in Statistics - Theory and Methods, 45(14): 4237–4248.

[3] Aliyev, R., Bayramov, V. (2017). On the asymptotic behaviour of the covariance function of the rewards of a multivariate renewal–reward process. Statistics and Probability Letters, 127: 138–149.

[4] Athreya, K.B., McDonald, D., Ney, P. (1978) Limit theorems for semi-Markov processes and renewal theory for chains. Ann. Probability, 6: 788–797.

[5] Gut, A. (1988) Stopped random walks. Limit theorems and applications. Springe, New York.

[6] Kesten, H. (1974) Renewal Theory for Functionals of a Markov Chain with General State Space. The Annals of Probability, 2, 3: 355–386.

[7] Lai, T. Siegmund, D. (1977) A non-linear renewal theory with applications to sequential analysis I. Ann. Statist, 5: 946–954.

[8] Lalley, S. (1982) Non-linear renewal theory for lattice random walks. Seq. Anal. 1: 193–205.

[9] Melfi, V.F. (1992) Nonlinear Markov renewal theory with statistical applications. The Annals of Probability, 20 (2): 753–771.

[10] Melfi, V.F. (1994) Nonlinear renewal theory for Markov random walks. Stochastic Processes and their Applications, 54: 71–93.

[11] Novikov, A.A. (2008) Some remarks on distribution of the first passage time and optimal stop of AR (1)-sequences. Theoriya veroyatn i ee primen. 53 (3): 458–471.

[12] Rahimov, F.H., Ibadova, I.A., Khalilov, V.S. (2019) Limit theorems for a family of the first passage times of a parabola by the sums of the squares autoregression process of order one (AR (1)). Uzbek Mathematical Journal, №2: 81–88.

[13] Rahimov, F.H., Ibadova, I.A., Hashimova, T.E. (2020) Limit theorem for first passage times in the random walk described by the generalization of the autoregression process. Uzbek Mathematical Journal, 4: 102–110.

[14] Saadatmand, A., Nematollahi, A. R., Sadooghi-Alvandi, S. M. (2016) On the estimation of missing values in AR(1) model with exponential innovations. Communications in Statistics - Theory and Methods, 46(7): 3393–3400.

[15] Shiryaev, A.N. (1996) Probability. Springer.

[16] Shiryaev, A.N. Essentials of Stochastic Finance: Facts, Models, Theory. Word Scientific, 2003, 843 p.

[17] Woodroofe, M. (1982) Nonlinear renewal theory in sequential analysis. SIAM. Philadelphia.

[18] Zhang, C.H. (1988) A Nonlinear Renewal Theory. Annals of Probability, 16(2): 793–824.

[19] Zhang, Y., Yang, X. (2010) Limit Theory for Random Coefficient First-Order Autoregressive Process. Communications in Statistics - Theory and Methods, 39(11): 1922–1931.

[20] G. Eason, B. Noble, and I. N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Phil. Trans. Roy. Soc. London, vol. A247, pp. 529–551, April 1955. *(references)*

[21] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68–73.

[22] I. S. Jacobs and C. P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G. T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271–350.

[23] K. Elissa, "Title of paper if known," unpublished.

[24] R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.

[25] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interface," IEEE Transl. J. Magn. Japan, vol. 2, pp. 740–741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].

[26] M. Young, The Technical Writer's Handbook. Mill Valley, CA: University Science, 1989.