Formulation of the Problem and General Requirements for Optimization Methods of Operational and Organizational Control Systems

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***Abstract*−The problem statement for the optimization of operational and organizational control systems is formulated and the general requirements for methods of solution are given. A probabilistic and deterministic approach to the problem of operational and organizational control taking into account the time factor is considered.**

***Keywords*−control systems, optimization methods, operational control, problems of synthesis**

I. Introduction

The term "operational and organizational control" in this paper means the processes of control and decision-making in organization systems and operational control systems based on the use of the prediction principle with periodic adjustments based on the processing of current information, i.e., rolling plan principle. Systems of this type find application in automated control systems at all hierarchy levels, are widely used to improve efficiency in all segments of industrial production, to optimize the control of vehicles, urban economy of large industrial centers, etc. Such operational control systems are successfully used to solve emergency management tasks [1]. The specifics of such tasks are associated with the need for an operational, i.e., prompt control decision making, and reflects the fact of time shortage. The time resource for control decision making in most cases is not constant, but depends on the current situation. Therefore, it is difficult to formalize mathematically correctly within the framework of the general formulation of the control problem. As a result, the design engineer has to rely on intuition or perform laborious and inefficient enumeration. In this paper, this problem is solved using the principle of complexity.

The term "operational and organizational control" reflects the two most significant features of the investigated control processes. The first of them is that the mathematical formalization of control problems of the control objects has constraints that are complex in form and various in content and do not fit entirely in the framework of known optimization problems, which requires taking into account a number of organizational measures. The second feature has to do with the need for operational, i.e., prompt control decision making and reflects the fact of time shortage.

The development of methods suitable for use in operational and organizational control systems has so far been carried out almost independently in the areas of optimal control theory, mathematical programming, operations research and artificial intelligence. We have made an attempt to classify all the numerous methods of approximate solution by their functional characteristics, reflecting the factors of taking into account the mathematical essence of the problem.

Problems of operational and organizational control are extremely widespread in the field of control, both for traditional dynamic objects and objects, the problems of optimal control of which have only begun to be studied in recent decades. These include, first of all, large transport networks, production sites of industrial enterprises, territories and facilities subject to catastrophic natural or man-made impacts, etc. To these objects, we can add large territorial associations, large information systems and, finally, computational processes and computing systems [2].

The optimization problems and methods investigated in this paper are widely used in all the above-mentioned areas of control. However, since the principles of their construction have certain peculiarities, the expediency of using operational and organizational control systems must be determined by comparison based on technical, operational and economic indicators with control systems of other types, such as systems based on the traditional principles of the automatic control theory [3]. At the same time, there is a fairly wide class of control objects in virtually all spheres of practical applications, where operational and organizational control systems are, apparently, the only means of achieving the set goal of control.

II. ORMULATION OF THE OPERATIONAL AND ORGANIZATIONAL CONTROL PROBLEM

Let us consider a control object characterized at each time instant t by a vector of state (output) parameters $\vec{x}\,(t)$ (this vector is also often called a phase vector), a vector of control variables $\vec{u}\,(t)$, a vector of disturbances t, and a vector of observable variables $\vec{y}(t)$. The latter are those generalized coordinates of the control object, information about the change of which is sent to the control system [4]. If the state parameters x^j can be changed directly, then $x^j = y^j$. Further, for simplicity, we assume that this condition is satisfied for all j and $\vec{x}\,(t) = \vec{y}\,(t)$.

The relationship between the vectors $\vec{x}\,(t), \vec{u}\,(t), \vec{\xi}\,(t)$ is given by the constraint equations

$$\vec{x}\,(t) = X\left[\vec{x}\,(t), \vec{u}\,(t_0, t), \vec{\xi}\,(t_0, t)\right], \qquad (1)$$

where t_0 is the initial time instant.

If the object can be characterized by differential equations, then instead of (1) we can write equations in the normal Cauchy form

$$\dot{\vec{x}}\,(t) = f\left(\vec{x}\,(t), \vec{u}\,(t), \vec{\xi}\,(t), t\right). \qquad (2)$$

To find $\vec{x}\,(t)$ from the known $\vec{u}\,(t)$ and $\vec{\xi}\,(t)$ on the time interval $[t_0, T]$, it is necessary to know the boundary conditions at its left end, i.e., $\vec{x}\,(t_0)$, or at the right end, i.e., $\vec{x}\,(T)$. In this case, the solution of equations (2) is called the solution of the Cauchy problem. In the case when it is necessary to transfer the object from the initial state $\vec{x}\,(t_0)$ to the final state $\vec{x}\,(T)$, i.e., both $\vec{x}\,(t_0)$ and $\vec{x}\,(T)$ are given, we say that the so-called two-point boundary value problem (or simply a boundary value problem) needs to be solved. The solution of the latter is much more challenging than the solution of the Cauchy problem.

Most practically important control problems include phase constraints (or constraints along the trajectory)

$$\vec{x}\,(t) \in E_x \qquad (3)$$

and control constraints

$$\vec{u}\,(t) \in E_u, \qquad (4)$$

where E_x and E_u are given varifolds in linear metric spaces R^{n+1} and R^{m+1}, respectively (n is the dimensionality of $\vec{x}$; m is the dimensionality of $\vec{u}$).

The choice of the vector function $\vec{u}\,(t)$ must comply with the requirement of the indicator of the control goal, given in the form of the following functional, reaching extremum:

$$E^*_{\text{id}} = E_{\text{id}}\left(\vec{x}\,(t), \vec{u}\,(t), \vec{\xi}\,(t)\right) \to extr. \qquad (5)$$

Reducing expression (5) to the extremum with constrain equations (1) or (2) and constraints (3), (4) in the general case is a difficult-to-solve problem. This is primarily due to the difficulties in obtaining equations (1) or (2) of the object presented to the control problems, as well as the difficulties in the practical implementation of the control law found for such a description of the object. Therefore, the implementation of the optimal control system, as a rule, is carried out as a result of the following two stages:

1) primary (ideal) optimization, when a simplified mathematical description of the control object is used to solve the variational problem; the vector control function found at this stage will be denoted by $\vec{u}_{\text{id}}\,(t)$, and the dependence $\vec{u}_{\text{id}}\,(t) = \vec{u}_{\text{id}}\,(\vec{x}\,(t), t)$ will be called the ideal control algorithm;

2) secondary optimization (optimization of control performance), which consists in finding such a realizable vector control function $\vec{u}^*\,(t)$, which, firstly, in the given sense differs minimally from $\vec{u}_{\text{id}}\,(t)$ and, secondly, takes into account the properties of the object to the degree of completeness, which is sufficient for the implementation of the set control goal.

3) As a measure of the approximation of an ideal control algorithm to the optimal one, functionals can be used, which connect:

4) extrema of the control goal achieved by the ideal and real control algorithm

5) $Q = Q\,[E^*_{\text{id}} - E^*]$;

6) ideal and real state vector

7) $\Phi = \Phi\,[\vec{\varepsilon}_{\text{id}}\,(t) = \vec{x}_{\text{id}}\,(t) - \vec{x}\,(t)]$.

8) The functional Q is called the control performance index, Φ – the accuracy factor, $\vec{\varepsilon}_{\text{id}}\,(t)$ – the error vector.

9) The use of the functional Q in comparison with the functional Φ is associated with much greater difficulties, which is due to the need to calculate the current values $E^*_{\text{id}}(t) - E^*\,(t) = \Delta E\,(t)$ and the possible ambiguity of the inverse dependence $\vec{u}\,(t) = \vec{u}\,(\Delta E\,(t), \vec{x}\,(t), t)$. For this reason, the accuracy factor Φ is used in practice. The diagram in which the functional Φ is used to form the control law in an automated system is shown in Fig. 1.

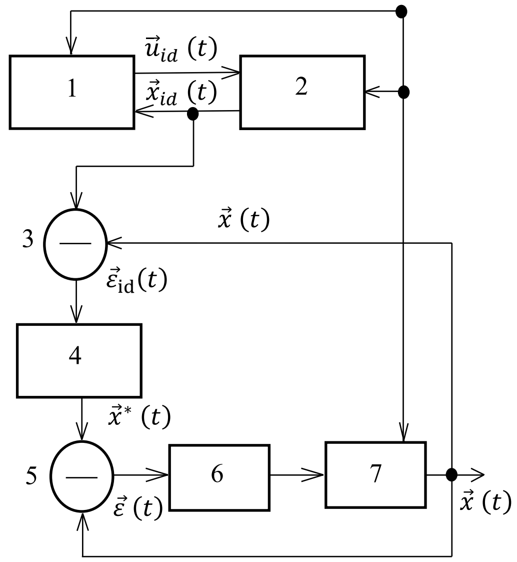

Fig. 1. The flowchart of an automatic control system based on the formation of an error signal from the setting vector $\vec{x}_{\text{id}}\,(t)$

Object *7* is influenced by uncontrolled impacts $\vec{\xi}\,(t)$, which in one form or another are taken into account by ideal model *2* formed on the basis of a simplified mathematical description of the object in form (1) or (2). Computer of ideal mode *1* solves the variational problem, the result of which results is $\vec{u}_{\text{id}}(t)$ and $\vec{x}_{\text{id}}(t)$. Comparison of $\vec{x}_{\text{id}}(t)$ and $\vec{x}(t)$ by device *3* allows us to determine $\vec{\varepsilon}_{\text{id}}(t)$, after which control accuracy optimizer *4* minimizes the functional Φ and calculates the optimal value of the state vector $\vec{x}^*(t)$. Comparator *5* calculates the difference $\vec{x}^*(t) - \vec{x}(t)$, which generates an error signal in a closed control circuit, which includes control object *7* and power amplifier *6*, which is usually necessary to implement control.

As a rule, a functional of the following form is chosen as the accuracy factor Φ:

$$\Phi = \int_0^{T_y} \varphi\{\vec{\varepsilon}_{\text{id}}(t)\}\,dt, \tag{6}$$

where φ is some continuous function of the arguments $\varepsilon_{\text{id}}^1(t), \varepsilon_{\text{id}}^2(t), \dots, \varepsilon_{\text{id}}^n(t)$; T_y is the moment of the end of object control.

In cases where the measure of the total deviation of the actual state vector $\vec{x}(t)$ from the ideal over a sufficiently large time interval is of interest, the integration limits in (6) are replaced with the boundary values of the interval $[-\infty \div +\infty]$. Sometimes, instead of (6), an estimate is used not on the interval $[0, T]$, but only at a certain instant $t = T$, i.e.,

$$\Phi = \varphi\,[\vec{\varepsilon}_{\text{id}}(T)]. \tag{7}$$

Obviously, it may turn out to be useful not to use functional (6) or a single estimate (7) as a control accuracy factor, but a set of estimates carried out at discrete instants t_k, which must be selected taking into account the specifics of the object and the set control goal.

Suppose the control object characterized by equations (1) must operate on the time interval $[t_0, T_y]$, and $(T_y - t_0) \leq \infty$. At the instant t_0, $\vec{x}(t_0)$. is known. Let us choose the interval $[t_0, T]$, assuming $(T - t_0) \ll (T_y - t_0)$, and for this interval on an accelerated time scale, using the predictive ideal model, we determine the ideal vector functions of state and control $\vec{x}_{\text{id}}(t)$ and $\vec{u}_{\text{id}}(t)$. Since $\vec{u}_{\text{id}}(t)$, which is a solution to the variational problem (1), (3)-(5), is usually unrealizable, we find the optimal control vector $\vec{u}^*(t)$ for $t \in [t_0, t_0 + T]$. For this purpose, the accuracy factor is also minimized on an accelerated time scale:

$$E = \int_{t_0}^{t_0+T} F\{\vec{x}_{\text{id}}(t), \vec{x}(t), \vec{x}(t_0), \vec{u}(t)\}\,dt$$

Under connection constraints, control constraints and phase constraints of the type (1), (3) and (4), respectively, but taking into account the conditions of realizability of the control.

We will implement control $\vec{u}^*(t)$ on the interval $\Delta t \ll T$. In this case, the control system will operate in an open loop up to the instant $t_1 = t_0 + \Delta t$. At the instant t_1, it is necessary to estimate the actual state of the object $\vec{x}(t_1)$ and calculate $\vec{x}_{id}(t)$ and $\vec{u}_{\text{id}}(t)$ for the interval $[t_1, t_1 + T]$ and the initial state $\vec{x}_{id}(t_1) = \vec{x}(t_1)$ again on an accelerated time scale. Next, it is necessary to develop an optimal control $\vec{u}^*(t)$ and implement it on the object during the time interval $[t_1, t_1 + \Delta t]$. Upon its expiration, i.e., at the instant $t_2 = t_1 + \Delta t$, $\vec{x}(t_0)$ is re-estimated, $\vec{x}_{id}(t)$ and $\vec{u}_{\text{id}}(t)$ are found, etc. By the instant $t_3 = t_2 + \Delta t$, this procedure is repeated and then resumes every time at discrete instants $t_k = t_{k-1} + \Delta t$ until the entire time interval $[t_0, T_y]$ has been passed. Systems that implement this principle of construction will be called operational and organizational control systems.

The first problem solved according to this principle was probably the problem of using the water of the reservoir for irrigation investigated by N.N. Moiseyev [5, 6]. He also proposed naming this principle (sometimes the word "approach" or "method" is used) the sliding plan principle. This term is especially useful when studying problems in the field of economics, but a special explanation of the term "plan" is required for many technical applications. Therefore, along with the traditional term, we will further use the term "operational and organizational control".

Thus, the operational and organizational control system operating on the time interval $[t_0, T_y]$, at the given discrete time instants $t_0, t_1 = t_0 + \Delta t, t_2 = t_1 + \Delta t, \dots, t_k = t_{k-1} + \Delta t, \dots$, firstly, predicts the ideal vector-function of state $\vec{x}_{\text{id}}(t)$ and control $\vec{u}_{\text{id}}(t)$, using the actual values of the state vector $\vec{x}(t_k)$ fixed at the time instants $\vec{x}(t_k)$ the initial conditions for the prediction; secondly, it predicts the optimal control $\vec{u}^*(t)$, which is the solution to the variation problem

$$E = \int_{t_k}^{t_k+T} F\,[\vec{x}_{\text{id}}(t), \vec{x}(t), \vec{x}(t_k), \vec{u}(t)]\,dt \rightarrow min;$$

$$\vec{x}(t) = \vec{x}\left[\vec{x}(t_k), \vec{u}(t_k, t), \vec{\xi}(t_{k,t})\right]; \tag{8}$$

$$\vec{x}(t) \in E_x;$$

$$\vec{u}(t) \in E_u.$$

and, thirdly, it implements control $\vec{u}^*(t)$ on the object during the time interval $[t_k, t_k + \Delta t]$.

The operational principle of such a system is illustrated by Fig. 2.

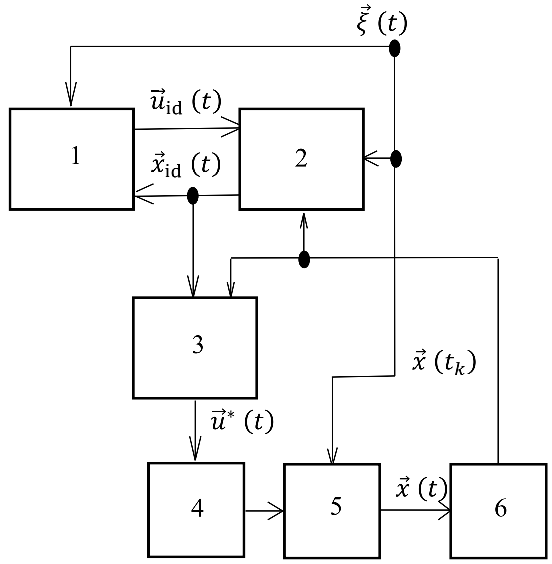

Fig. 2. The structural principle of an operational and organizational control system: 1 – computer of ideal mode; 2 – predictive ideal model; 3 – computer of $\vec{u}^*(t)$; 4 – actuating elements; 5 – object; 6 – meter of $\vec{x}(t_k)$

Let us call the time interval $[t_k, t_k + T]$, on which it is required to find the ideal and optimal controls, the scheduling interval, and the interval $[t_k, t_k + \Delta t]$ the scheduling repetition interval. These intervals can vary for the same object. The main considerations for their selection will be given in the following paragraphs.

It should be emphasized that for some types of control objects, there may be no need to solve the second variational problem with criterion (8). Indeed, if mathematical model (1) of the object and constraints (3) and (4) characterize the object sufficiently completely, then we can assume $\vec{x}_{\mathrm{id}}(t) = \vec{x}^*(t)$ and $u_{\mathrm{id}}(t) = \vec{u}^*(t)$. In this case, there will be no block *3* in the diagram shown in Fig. 2, and the input of block *4* will be the signal $\vec{u}^*(t) = \vec{u}_{\mathrm{id}}(t)$.

It is advisable to consider some examples for a more visual explanation of the structural principle of an operational and organizational control system.

III. PROBLEMS OF OPTIMAL SYSTEM SYNTHESIS AND OPTIMIZATION OF OPERATIONAL AND ORGANIZATIONAL CONTROL

The design of modern optimal automatic control systems, as a rule, is performed in two stages: 1) calculation of the optimal programmed (reference or nominal) trajectory of the object's motion; 2) calculation of the optimal controller or the closed-loop system of optimal control proper, whose objective includes the optimal, in the given sense, approximation of the real motion of the object to the programmed trajectory [7]. This practice of division of tasks is a result of many years of engineering experience in the design of control systems. Exceptions are observed in two cases: when there is no need to calculate the optimal programmed motion, which occurs in many problems of the theory of automatic control, where the programmed mode is set unambiguously, for instance, in the form of a given engine speed; when the problem of designing the optimal system is so simple that both stages can be performed simultaneously. Unfortunately, the latter case is extremely rare in the practice of modern control system design.

Solving a common problem in two stages is a necessary measure due to the difficulty, and in most cases, the impossibility of combining both stages. However, in some cases, for instance, when the optimal system is synthesized according to the criterion of minimum energy costs and energy consumption for motion along the programmed trajectory is much higher than the costs for keeping the object near the programmed trajectory, separate handling of problems is mathematically justified. This means that the obtained optimal solution will be close to that which would give a combined solution to the problem of selecting a programmed trajectory and calculating the optimal controller. For other, more general cases, a proof of the possibility of separating tasks is required. In real-life design, this proof is often given in the form of engineering intuition and experience, because a rigorous mathematical solution to the problem has not yet been obtained.

The first stage of designing the optimal control system, i.e., the development of an optimal programmed trajectory (or just a program) is the simplest one. This is due to two main reasons. First, at this stage, we can consider simpler equations of the object's dynamics, which do not take into account a number of factors. Second, when calculating the programmed trajectory, it is sufficient to define the control law of the object as a function of time and boundary conditions (the initial and final state of the object).

When calculating the optimal controller at the second stage, it is necessary to find the optimal control as a function of the state of the object. These two problems have different physical and mathematical content.

In the problems of determining optimal programmed motion, the dynamics of an object is given by differential equations of the type (2) or, in a deterministic formulation, by the equations

$$\dot{\vec{x}} = \vec{f}(\vec{x}, \vec{u}, t), \qquad (9)$$

where, as before, $\vec{x}$ is an *n*-dimensional state vector; $\vec{u}$ is an *m*-dimensional control vector.

The solution of equations (2) or (9) on the interval $[t_0, T_y]$ must ensure the extremum of the selected indicator of the control goal [denote it by $E_1(\vec{x}, \vec{u})$], satisfy the boundary conditions at the left, at the right, or at both ends of the trajectory: $\vec{x}(t_0) = \vec{x}_0, \vec{x}(T_y) = \vec{x}_r$ and be subjected to constraints (3) and (4).

Suppose the problem of selecting the optimal programmed trajectory has been solved by one of the methods and the vector functions $\vec{u}^*(t)$ and $\vec{x}^*(t)$ have been found for $t \in [t_0, T_y]$. Here, the problem of *optimal system analysis* is said to have been solved. However, the real motion of the object due to the influence of uncontrolled disturbances, the impossibility of absolutely accurately setting the object in its original initial state and absolutely accurately realizing the optimal programmed control $\vec{u}^*(t)\vec{u}^*(t)$, etc. will differ from the programmed one. If we introduce the notation

$$\vec{z}(t) = \vec{x}(t) - \vec{x}^*(t);$$

$$\vec{v}(t) = \vec{u}(t) - \vec{u}^*(t),$$

where $\vec{x}(t)$ is the trajectory of the real motion of the object during the control $\vec{u}(t)$; $\vec{v}(t)$ is an additional control, bringing the real motion closer to the programmed one, then it is possible to obtain the equations of the relative motion of the object during the control

$$\dot{\vec{z}}(t) = \vec{f}(\vec{x}, \vec{u}, \vec{\xi}, t) - \vec{f}(\vec{x}^*, \vec{u}^*, t).$$

The last equation, under the assumption that the deviation of the real trajectory from the programmed one is small, can be represented as

$$\dot{\vec{z}}(t) = \vec{f}(\vec{z}, \vec{v}, \vec{\xi}, t) \qquad (10)$$

or, if linearization is acceptable, in the form

$$\dot{\vec{z}}(t) = A\vec{z}(t) + B\vec{v}(t) + \vec{\xi}(t), \qquad (11)$$

where *A* and *B* are matrices with constant or variable coefficients; $\vec{\xi}$ is a disturbance vector.

The presence of the component $\vec{\xi}(t)$ in equations (10) or (11) leads to the need to formulate complex optimal control problems that are associated with minimizing the mathematical expectation of some criterion $E_2(\vec{z}, \vec{v})$ and are currently far from the final solution. The results available in this area can be found in [5, 6]. In the following paragraphs, we will consider deterministic problems of optimal control, when the components $\xi\ (t)$ in equations (10) and (11) can be neglected.

Obviously, to calculate the optimal controller that controls the real motion of the object, it is not enough to find the relationship between the control and time, i.e., $\vec{v}\ (t)$, this requires determining the relationship between the control and the coordinates of the object and time $\vec{v}\ (\vec{z}, t)$. This dependence has to be optimal, i.e., to minimize the value of the selected functional $E_2\ (\vec{z}, \vec{v})$. The functionals $E_1\ (\vec{x}, \vec{u})$ and $E_2\ (\vec{z}, \vec{v})$ can have both the same and different physical meanings. Generally speaking, when calculating the optimal controller, two different approaches are possible. The first approach is when the controller is created in such a way that when a misalignment occurs between the programmed and real trajectories, the controller must return the object to the programmed trajectory. For instance, for any type of the criterion $E_1\ (\vec{x}, \vec{u})$, the controller can be constructed in such a way as to minimize the time of the object's return to the programmed trajectory. In this case, the physical meaning of $E_1\ (\vec{x}, \vec{u})$ and $E_2\ (\vec{z}, \vec{v})$ is different. The second approach is based on the construction of a new programmed trajectory relative to the real state of the object and motion along this trajectory. This approach is called correction according to a given program. Obviously, in the second case, the criteria $E_1\ (\vec{x}, \vec{u})$ and $E_2\ (\vec{z}, \vec{v})$ have the same physical meaning.

Based on the above, the formulation of the problem of calculating optimal correcting controls will look as follows: find the control $\vec{v}^*(\vec{z}, t)$ as a function of phase coordinates and time minimizing the functional $E_2\ (\vec{z}, \vec{v})$ and satisfying equations of relative motion of the type (10) or (11).

The solution to this problem is called *optimal control synthesis*. Since determining $\vec{v}\ (\vec{z}, t)$ in most cases presents serious mathematical difficulties, we have to simplify the problem and look for $\vec{v}\ (\vec{z}, t)$ in some narrower class of admissible controls. In this case, we speak of the solution of the problem of possible or virtual synthesis [5, 6]. However, knowing the optimal controls $\vec{v}\ (\vec{z}, t)$ turns out to be insufficient for the design of an optimal control system. To do this, it is necessary to implement a controller that provides feedback, which in mathematical terms comes down to determining the operator W, which (e.g., in the linear case) has the form

$$W = a_k \frac{d^k}{dt^k} + \cdots + a_1 \frac{d}{dt} + a_0$$

and links the variables v and z: $v = Wz$. Such a problem is called *optimal system synthesis*. Its solution is a much more complicated problem than finding the optimal controls $\vec{v}\ (\vec{z}, t)$.. Formally, in the general case, it is reduced to nonlinear programming problems in function spaces. With the introduction of simplifications, e.g., when approximating W by finite-dimensional functions, it turns out to be possible to proceed to simpler problems - the problem of nonlinear programming in a finite-dimensional space or to the problem of optimal control synthesis [5, 6, 8].

There is another noteworthy fact. Suppose that equations (2) describe the dynamics of the object sufficiently completely. Then the solution to the synthesis problem will consist in finding a control $\vec{u}^*\ (\vec{x}, t)$ depending on the phase coordinates and time such that turns the criterion $E_1\ (\vec{x}, \vec{u})$ into an extremum, satisfies constraint equations (2), the boundary condition at the right end and constraints (3), (4).

If, in the problems of optimal system synthesis, we compare the principle of correction according to the final state with the principle of constructing systems of operational and organizational control, it is easy to find an analogy. Indeed, determining the optimal control $\vec{u}^*\ (t)$ on the interval $[t_0, T_y]$ with the help of operational and organizational control systems and realizing it during the time Δt, we obtain the state $\vec{x}\ (t_0 + \Delta t)$ different from $\vec{x}^*\ (t_0 + \Delta t)$. Next, an optimal control is generated with respect to the state $\vec{x}\ (t_0 + \Delta t)$ on the planning interval $[t_0 + \Delta t,\ T_y]$, at the time instant $t_0 + 2\Delta t$, a control is generated with respect to $\vec{x}\ (t_0 + 2\Delta t)$ on the interval $[t_0 + 2\Delta t,\ T_y]$, etc., up to the end time instant.

Two important conclusions can be drawn from a comparison of problems of optimal system synthesis and problems of operational and organizational control.

The implementation of the system of operational and organizational control does not require solving the problem of optimal control synthesis. The approximation of the real motion of the object to the optimal one is achieved by repeatedly solving the problem of developing optimal programs (analysis problems).

The use of operational and organizational control systems, just like the end-state correction systems, in a number of practically important cases can give significant technical or economic advantages in comparison with systems in which correction according to the given program is used [5, 6]. Let us illustrate this with the following example. Let it be required to solve the problem of ensuring the maximum range of the rocket with a given fuel supply. Considering the rocket as a material point, we can find the optimal trajectory of its motion. Due to a number of conditions mentioned earlier, the real motion will differ from the optimal one. Let us assume that as a result of the action of factors of a random nature, it turned out to be possible to achieve a greater range than expected. These circumstances should be used to the maximum, without worrying about the real trajectory having to be close to the optimal one. Obviously, when correcting according to the given program, it is impossible to use these circumstances.

IV. Operational and Organizational Control Systems Operating on the Principle of Approximation to a Given Program

Control systems based on the principle of correction according to a given program can have certain advantages over systems with end-state correction, which is mainly ensured by a simpler implementation of such systems. If the object under consideration indeed has such advantages, it is advisable to use them. We will show how this can be done when building a operational and organizational control system. Let us consider the problem in a deterministic formulation.

Let the dynamics of the object be characterized by equations of the type (2), but in the absence of the components $\vec{\xi}(t)$; boundary conditions at the right and left ends, constraints (3), (4) and criterion $E_1\ (\vec{x}, \vec{u})$ are given. Thus, we need to find a solution to the following problem:

$$E_1\ (\vec{x}, \vec{u})\ \rightarrow min;$$

$$\dot{\vec{x}} = \vec{f}\ (\vec{x}, \vec{u}, t);$$

$$\vec{x}\,(t_0) = \vec{x}_0; \quad (12)$$

$$\vec{x}\,(T_y) = \vec{x}_T;$$

$$\vec{x}\,(t) \in E_x;$$

$$\vec{u}\,(t) \in E_u.$$

Let us denote, as before, the optimal control found as a result of solving problem (12) by $\vec{u}^*\,(t)$,, and the optimal trajectory by $\vec{x}^*\,(t)$. The found vector functions $\vec{u}^*\,(t)$ and $\vec{x}^*\,(t)$ are obtained for the initial condition $\vec{x}\,(t) = \vec{x}_0$. Let us expand the region of possible initial states and solve problem (12) for each state in this region. As a result, a set of optimal trajectories will be obtained. We find in this set the trajectory that corresponds to the smallest value of the criterion $E_1\,(\vec{x},\vec{u})$ and call it ideal. In Fig. 3 for the set of initial states bounded by the segment *AB*, the optimal and ideal trajectories are denoted by $x^*\,(t)$ and $x^{**}\,(t)$, respectively.

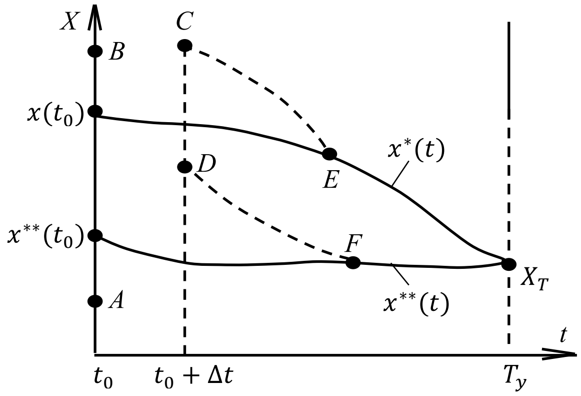

Fig. 3. Optimal $x^*\,(t)$ and ideal $x^{**}\,(t)$ trajectories

When implementing the operational and organizational control system, as a result of the action of the control $u^*(t)$, the object will go from the state $x\,(t_0)$ to the state $x\,(t_0 + \Delta t)$ after the time interval $[t_0,\ t_0 + \Delta t]$. Let this state be characterized by the point *C* (see Fig. 3). The calculation of the optimal program relative to the obtained state gives the trajectory of *CE* transferring the object to the given trajectory $x^*(t)$. If, as a result of the action of uncontrolled disturbances, the real state of the object is characterized by the point *D* (see Fig. 3), then the solution of problem (12) with respect to the new state and the time instant $t_0 + \Delta t$ can give a trajectory that transfers the object to the ideal trajectory $x^{**}\,(t)$, i.e., trajectory *DF*. Thus, in cases where the optimal and ideal programmed trajectories do not coincide, the operational and organizational control system can develop trajectories that approximate either optimal or ideal. An alignment of trajectories for some initial states $x\,(t_0 + k\Delta t)$ can occur at the point x_T. Let us consider a simple example to illustrate this.

We have the following problem:

$$E\,(x,u) = \int_0^T (x^2 + u^2)dt + \lambda\,[x\,(T) - \beta]^2 \to min;$$

$$\dot{x} = u;$$

$$u \in E_u = [u: -2, -1, 0, 1, 2]; \quad (13)$$

$$x_{min} \le x\,(t) \le x_{max};$$

$$0 \le x(t) \le \beta.$$

For the parameters $T = 10$; $\lambda = 2{,}5$; $\beta = 2$; $x_{min} = 0$; $x_{max} = 8$ and a quantization step in time and in x equal to 1, using the computational procedure of dynamic programming a set of optimal solutions for $0 \le x\,(t_0) \le 8$ was obtained.

Thus, approaching problems of synthesis of optimal trajectories from the standpoint of operational and organizational control led to the finding a new method of correction — correction according to an ideal program. For further constructions, let us clarify the concept of correction. By correction we mean the development of control actions that return the object from a deviated trajectory to the given state, to a programmed or ideal trajectory so that the extremum of the required functional is provided. In a special case, such a correction will correspond to a correction that gives the fastest possible return to the reference or ideal trajectory.

Let us now formulate two statements on the basis of which optimization algorithms can be constructed in operational and organizational control systems.

Statement 1. In the set of possible states of the system in the vicinity of the reference trajectory, there may exist those for which correction according to an ideal program is more effective in the sense of the sought-for functional than correction according to a given program.

Statement 2. The choice between the two said correction methods can be made by dividing the set of possible states into two classes $M\,(x^*)$ and $M\,(x^{**})$, the former of which indicates the preference for correction according to a given program over correction according to an ideal program, the latter indicating the opposite.

This division is called zoning. For the considered scalar problem, it can be performed in the simplest way as follows:

$$M\,(x^*) = [x:\ \rho\,(x, x^{**}) > \rho\,(x^*, x^{**})];$$

$$M\,(x^{**}) = [x:\ \rho\,(x, x^{**}) < \rho\,(x^*, x^{**})].$$

where x is the coordinate of the real trajectory; $\rho\,(a,b)$ is the distance between a and b.

Obviously, there is a quite wide class of problems for which: 1) $M\,(x^*) \cup M\,(x^{**})$ forms the entire set of possible states, i.e., corrections according to $x^*(t)$ and $x^{**}(t)$ solve the optimal synthesis problem; 2) corrections according to $x^*(t)$ and $x^{**}(t)$ are equivalent to corrections according to the criterion of maximum performance.

Statements 1 and 2 make it possible to simplify the algorithms for optimizing operational and organizational control, in which, as mentioned earlier, the end-state correction is used. In this case, the optimization process will consist in the following:

1) determining $x^*(t)$ and $x^{**}(t)$, here the function $x^{**}(t)$ can be obtained at the stage of preliminary calculations;

2) determining if concrete values of $x\,(t)$ belong to the class $M\,(x^*)$ or $M\,(x^{**})$;

3) implementation of correction according to $x^*(t)$, $x^{**}(t)$ or end state depending on the results of Step 1.

With an appropriate choice of the optimization method, the calculation of $x^*(t)$, $x^{**}(t)$ and correcting controls can be carried out using one algorithm.

V. PROBABILISTIC AND DETERMINISTIC APPROACH TO THE PROBLEM OF OPERATIONAL AND ORGANIZATIONAL CONTROL

The purpose of building control systems that have the structure and principle of operation of operational and organizational control systems is not only to overcome the difficulties associated with solving synthesis problems, but also the difficulties caused by the need to take into account random disturbances acting on the control object (difficulties of stochastic synthesis). There are significant possibilities for simplifying the calculation methods in the very principle of constructing such systems, which is associated with the periodic correction of previously developed control actions based on taking into account the actual state of the controlled process. Indeed, in those cases when the influence of random factors on the planning repetition interval turned out to be such that the actual state slightly differs from the calculated one, the development of correcting controls can be carried out based on the deterministic models of the object. Thus, for objects of this type, random disturbances can only be taken into account by correcting control actions at appropriately selected time intervals. Consequently, the choice of the planning repetition interval will determine the accuracy provided by the operational and organizational control system. The smaller this interval, the greater the accuracy, but the higher the requirements for the computing capacity of the control system [9].

When using probabilistic methods, the optimal solution should be developed taking into account the statistical characteristics of the vector of random disturbances. A more complete account of the properties of the object will allow the correction to be carried out less frequently, i.e., to increase the planning repetition interval with the same accuracy of the control system. But at the same time, the difficulties in implementing probabilistic methods will increase very significantly. Obviously, when choosing one of the two alternatives - using a deterministic or probabilistic approach in the task of constructing an operational and organizational control system - it is necessary to take into account the ensured accuracy of the solution, the value of the planning repetition interval and the difficulty of implementing the method by the computational means of the system. Usually, these difficulties are crucial, since in the deterministic formulation the solution to the problem of developing optimal controls (programs) is much simpler than in the probabilistic formulation.

It should be emphasized that the use of a deterministic approach, on which all further presentation of the material will be focused, does not at all mean a complete neglect of random disturbances acting on the control object in the process of calculating optimal controls. These disturbances are taken into account when constructing the model of the control object. The easiest way to do this is: 1) when constructing models in which the vector of the object's state is characterized by its mean values or mathematical expectation; 2) when constructing models with predicted values of the components of the state vector.

In the first case, when developing models, the moving average method can be effectively used [1], in the second case, predictive devices of various kinds (see, e.g., [3]).

In the process of analyzing the control object, it can be found that the level and nature of the action of random disturbances is such that neither an increase in the frequency of correction of control actions, nor the improvement of predictive devices can ensure the specified accuracy of the implementation of operational and organizational control systems. In these cases, the use of a deterministic approach is hardly justified. It is necessary to study the possibilities of probabilistic methods and other principles of building a control system.

Comparison of the results of stochastic synthesis with the results of operational and organizational control, which is an issue of natural interest, is currently not fully resolved. However, in [8], a theoretical analysis of the results of stochastic synthesis, piecewise-deterministic synthesis (at discrete instants in time, the disturbance prediction is carried out and the problem of deterministic synthesis is solved) and piecewise-programmed (operational and organizational) control for linear dynamic systems optimized by the quadratic functional. It is shown that in those cases when the disturbance predication is continuously corrected, i.e., the interval Δt is close to zero, the results of all the above control methods coincide both for the disturbance in the form of white noise and for the filtered white noise. In addition, the results of stochastic and piecewise deterministic synthesis coincide for any values of the planning repetition interval for disturbances in the form of white noise. For filtered white noise and prediction of disturbances at the instants $t_0 + k\Delta t$ the difference between the value of the functional for operational and organizational control and stochastic synthesis is non-negative and has the order Δt. Recall that here, in both cases, the functionals have the meaning of the mathematical expectation of the criterion used.

VI. OPTIMIZATION METHODS USED IN OPERATIONAL AND ORGANIZATIONAL CONTROL SYSTEMS AND THE REQUIREMENTS FOR THEM

Along with the term "operational and organizational control", the terms "operational control" and "organizational control" are widely used. In the latter case, we mean the control of the so-called organizational systems, i.e., systems that include objects of physical nature and teams of people. Systems of this type are also called large systems. The main definitions and problems standing in the way of managing large systems are outlined in [10]. It is important to emphasize here that the above principle of building control systems is widely used in building organizational systems. Examples include almost all problems associated with planning and taking action to carry out the plan.

In the term "operational and organizational control" the second word means not only the applicability of the principle of building such systems to the problems of managing organizational systems, but also control objects having certain specific features. These features are mathematically characterized by the presence of various kinds of logical conditions that must be taken into account in the control process. The presence of conditions of this type introduces additional features into the apparatus of mathematical programming methods used to solve the formulated problem of developing optimal controls. Sometimes these features are such that a standard, well-developed optimization method cannot be used, but sometimes they greatly facilitate the computation process.

The term "operational control" covers systems that have the same construction principle as operational and organizational control systems, however, the presence of specific features of a logical type is not reflected in the mathematical models of the object. Therefore, both these terms, if we do not emphasize the presence of logical conditions, can be considered synonymous.

The form of mathematical description of objects, for the control of which it is advisable to use operational and organizational control systems, may vary, but the presence of phase constraints (3) and control constraints (4), as well as conditions of a logical type, is an integral characteristic feature of such objects. This feature imposes certain requirements on the optimization methods used. Let us list the main requirements for optimization methods.

If the control object is described by differential equations, it is often necessary to solve a two-point boundary value problem. The only method that allows solving two-point boundary value problems without special laborious tricks is a computational procedure of dynamic programming.

Computational procedures of dynamic programming, as will be seen from the subsequent presentation, without significant modifications allow implementing both the deterministic and the probabilistic approach in operational and organizational control systems.

So, at present, considerable attention is paid to the development of methods for optimal control of dynamic systems under constraints on control of the integral form

$$\int_{t_0}^{T} \varphi\,(\vec{u})\,dt \ \leq C.$$

The use of the proposed approach for constructing a system and computational procedures of dynamic programming does not cause fundamental difficulties in solving this problem.

To overcome the dimensionality problem, numerous techniques and methods have been proposed, many of which are outlined below and analyzed for both general and specialized problems.

Finally, note another extremely important limitation imposed on optimization methods by the specifics of their use for operational and organizational control systems - an extremely limited resource of time allocated for the calculation of optimal controls. More on this in the next section.

VII. TIME FACTOR IN OPERATIONAL AND ORGANIZATIONAL CONTROL SYSTEMS

The principle of operation of operational and organizational control systems is based on a periodic assessment of the actual state of the object, the development of optimal control actions based on this assessment and their implementation in the object until the next assessment of the state. Naturally, the problem of selection of the optimal controls should be solved during such a time interval $\tau_p < \Delta t$, after which the meaning of the implementation of the obtained control still does not disappear, and it is natural that the interval τ_p should be sufficiently small.

In this case, is chosen the following procedure of the system operation. At each time instant $(t_k - \tau_p)$, the actual state $\vec{x}(t_k - \tau_p)$ is estimated and the state $\vec{x}(t_k)$ is predicted. Knowing $\vec{x}(t_k)$, the optimal values of the control actions $\vec{u}^*\,(t)$ are calculated. The calculation should be completed by the instant t_k. Starting from this time instant, the developed control is implemented up to the time instant $t_{k+1} = t_k + \Delta t$.

If it is possible to predict $\vec{x}(t_k)$ accurately enough, the interval τ_p can be taken large enough but still not exceeding the planning repetition interval Δt.

If for some reason it is not possible to carry out a qualitative prediction, the operation of the operational and organizational control system should be arranged in a different way. In this case, it is necessary to periodically perform the following steps: to estimate the state vector $\vec{x}\,(t)$ at time instants $t_k, t_{k+1}, t_{k+2}, \dots$ and select the optimal control $\vec{u}^*\,(t)$; to implement the developed control $\vec{u}^*\,(t)$ starting from the instant $t_k + \tau_p$.

Since the implementation of $\vec{u}^*\,(t)$ starts not from the instant t_k, but from the instant $t_k + \tau_p$, i.e., with the delay τ_p, additional errors may occur due to the fact that $\vec{x}\,(t_k)$ differs from $\vec{x}\,(t_k + \tau_p)$. Therefore, it is necessary to strive to make the interval τ_p as small as possible.

The problem of minimizing τ_p also arises for those operational and organizational control systems where the planning repetition intervals Δt are not constant and can even be random variables, and the state of the object during this time can change significantly.

Thus, the time factor when choosing an optimization method for operational and organizational control systems is very important and sometimes crucial. To obtain a solution in a timely manner, it is often necessary to completely abandon the choice of optimal controls and use approximate optimization methods, i.e., methods that give a suboptimal, but sufficiently close to optimal solution.

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