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New Approach to Calculation of Transition Curves on Curved Roads

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***Abstract*—The article sets forth a new method for compiling transition curves for projected and constructed highways at the entrance from straight sections to horizontal curves. The expression for the transition curve is found, as a special case, from the solution of the differential equation of curvature. To substantiate the results, a monotonic increase in centrifugal acceleration along the curve was revealed.**

Keywords—curvature, serpentine, centrifugal acceleration, dimensionless parameter, road, transition curve, particular solution, trajectory

When the car moves on the road, when entering from straight sections to horizontal curves (for example, zigzag roads and serpentines), the driving condition changes and the influence of centrifugal force on the car begins. In theory, this force acts instantly on the car at entry points, but practically when the steering wheel is turned from the driver's side within small areas. Observations show that, as a rule, when cars enter curves with a radius of less than 600 m, drivers reduce their speed. To avoid abrupt changes in traffic conditions and reduce the convenience of movement between straight and curved sections of highways, additional curves are plotted, called transition curves. The curvature of the road axis within these two curves gradually changes from 1 / ∞ to 1 / R [1].

According to the current standards for the design of motor roads, on all motor roads (on roads of I ... V category), when the radius of the curves is less than 2000 m, transition curves are necessarily projected. The presented article examines the reliability of the results obtained using the expression for the proposed transition curve on the example of projected roads.

Designing a road plan is considered the most important and crucial stage in all road design. Since, the amount of road construction costs, transportation and operating costs, the level of convenience, safety issues, etc. depend on it. Part of highways consists of transition curves of the transition from the straight part of the road to the curves. A transition curve is a curve that gradually changes the radius of curvature. When deriving the equation of the transition curve, it is necessary to take into account the change in the modes of movement of the vehicle and the force acting on the vehicle when driving along the transition curve, as well as all the requirements for the safety and comfort of road traffic.

According to the current standards for the design of highways, to ensure the safe movement of vehicles on curves in the plan and longitudinal profile of roads, the main condition is a small change and limitation of speed. When you change the category of the road, the speed of movement also changes, and depending on it, the minimum values of the radius of the curves are different (table 1).

Various methods are used to construct transition curves (clothoid curve, Bernoulli lemniscate, cubic parabola, etc.) [2]. In the presented article, for the construction of transition curves to the above methods, an alternative method is created and the differential equation of curvature is solved by the analytical method, as an inverse problem of a particular case in a Cartesian coordinate system [3-5,7-9].

- $k(x)$ curvature is selected based on the following criteria:
- the function $k(x)$ must be simple and integrable;

the function $k(x)$ in a given interval must have one extremum and the value of the functions $k(x)$ must be negative in all values of the argument for the curve to be convex;

- at the starting point it is better to have a smooth transition from a straight line to a curved line.

Table 1

Road category	Calculated speed, km / h	Minimum (main) radius of curvature in the plan, m.
Ia	150	1200
Ib	120	1000
II	120	800
III	100	600
IV	80	300
V	60	150
	50	100

For this, the condition at the origin of coordinates $k(x) = 0$ must be satisfied.

The curved line defining the curvature must be smooth and convex upward in the specified interval. For example, on the interval [0, π] (1):

$$k(x) = -\mathrm{a} \cdot \sin(px) \tag{1}$$

$$k(x) = \frac{\frac{d^2y}{dx^2}}{\sqrt{\left(1 + \left(\frac{dy}{dx}\right)^2\right)^3}} \tag{2}$$

when solving the differential equation of curvature (2) taking into account expression (1), the following expression (3) is obtained:

$$y_1(t) = \frac{1}{p} ln\left(\sin(pt) + \sqrt{B^2 - cos^2(pt)}\right) + C, \tag{3}$$

where: $B = p/a$; a is the maximum value of the curvature; p is the scale factor of the argument; t is a dimensionless parameter ($t = x/b$), the value of which varies in the interval [0,1].

X1, Y1 is the start point and X2, Y2 is the end point of the transition lines.

$C = -\ln(B^2 - 1)/2p$ is a constant, and is determined from the boundary conditions $X_1 = 0$, $Y_1 = 0$ (coordinates of the starting point of the transition curve) (Figure 1).

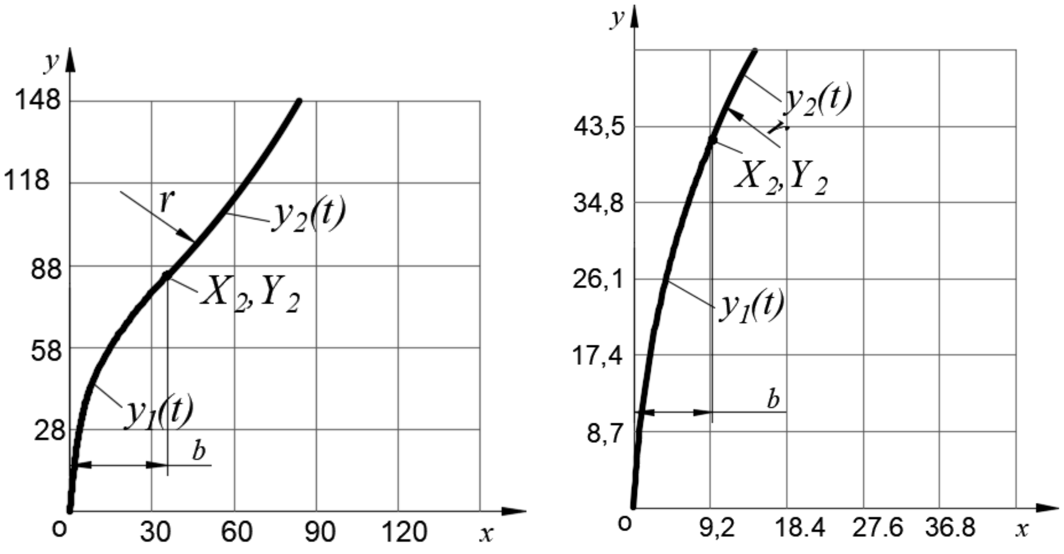
All dimensions are given in meters. To pass to a dimensionless quantity, all geometric

parameters are divided by b, where $b = |X2 - X1|$

$$x_r = \frac{X_r}{b}, \quad y_r = \frac{Y_r}{b}, \quad x_2 = \frac{X_2}{b}, \quad y_2 = \frac{Y_2}{b}, \quad r = \frac{R}{b},$$

The trajectory of a circular curve is expressed by the well-known circle formula:

$$y_2(t) = \sqrt{r^2 - (t - x_r)^2} + y_r, \tag{4}$$



a ***b***

Figure 1.a) Trajectory of the road: y1 (t) transition curve, y2 (t) circular curve,

$R = 100, X_r = -40, Y_r = 100$ (radius and coordinates of the center of the circular curve);

b) $R = 100, X_r = 103, Y_r = 11$ (radius and coordinates of the center of the circular curve).

At the point (x_2, y_2), the tangents of both curves must coincide. That is, this expression must be executed: $\frac{dy_1(x_2)}{dx} = \frac{dy_2(x_2)}{dx}$. To obtain this condition, x_2 is iterated over.

$$y_2(t) = \sqrt{r^2 - (t - x_r)^2} + y_r$$

Figure 1b shows another trajectory of the road. Here a part of the circular curve of the road is built according to the formula (4).

The magnitude of the tangent at the start and end points of the transition curve of the road is determined by the formula (5):

$$\frac{dy_1(t)}{dt} = \frac{\cos(p \cdot t)}{\sqrt{\left(\frac{p}{a}\right)^2 - \cos(p \cdot t)^2}}, \tag{5}$$

The value of the vehicle speed on the transition curve is considered constant. The centrifugal force increases in direct proportion to the transition time of the transition curve, that is, from the starting point at a distance *L* with a radius of curvature *R*, it reaches within τ (seconds) after the vehicle enters the transition curve:

$$\frac{V^2}{R} = J \cdot \tau \tag{6}$$

where τ is the time during which the car passes the transition curve of length *L; R* is the radius of curvature of the curve at a given point; v is the speed of movement and *J* is the rate of increase of centrifugal acceleration. The rate of increase of centrifugal acceleration is found as follows:

$$J = \frac{V^3 \cdot k(t)}{L(t)} \tag{7}$$

where k (t) is the coefficient of curvature at a given point on the curve.

In formula (7), the distance from the beginning of the curve to a given point - $L(t)$, is determined from expression (8) taking into account (5):

$$L(t) = \int_0^t \sqrt{1 + \left(\frac{dy1(t)}{dt}\right)^2} dt = \frac{p}{a} \int_0^t \frac{dt}{\sqrt{\left(\frac{p}{a}\right)^2 - \cos(pt)^2}} \quad (8)$$

In fig. 2 shows a graph of the dependence of the path traveled along the transition curve on the x coordinate.

To ensure comfortable and safe driving on roads, the minimum length of the transition curve should be determined, taking into account the condition of maintaining the permissible value of the increase in the speed of centrifugal acceleration of the vehicle [6]. In the road design standards of most countries, it is assumed that the value of J is in the range of 0.3 ... 1 m/s^3. In Azerbaijan, the maximum value of the rate of increase of centrifugal acceleration is considered to be equal to 0.8 m/s^3 (for high-speed roads).

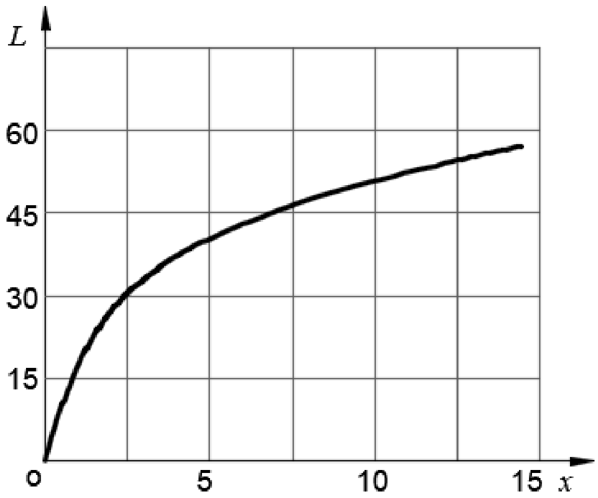


Figure 2. The path traversed by the transition curve

Coordinate x at any point of the road on the transition curve is determined from expression (8), and y - from expression (3). Table 2 shows the dependence of the coordinates of the transition curve on the distance traveled along the transition curve (Figure 1, b), calculated on the basis of expressions (3) and (8).

Table 2

Distance traveled	Coordinates	
L, м	X, м	Y, м
0	0	0
5	0,6454	4,9582
10	1,3118	9,9136
15	2,0211	14,8630
20	2,7961	19,8025
25	3,6618	24,7269
30	4,6456	29,6290
35	5,7783	34,4987
40	7,0941	39,3221
42,0988	7,7100	41,3285
45	8,6312	44,0794
50	10,4316	48,7432

The transition curve with the smallest length is determined by the condition of compliance with the standard values, the rate of increase of the centrifugal acceleration of the vehicle, as well as comfortable and safe movement on the road [6]:

$$l \geq \frac{V^3}{47 \cdot J \cdot R}, \quad (9)$$

where, V is the estimated speed of the vehicle, km / h; J is the standard value of the rate of increase of the centrifugal acceleration of the vehicle;

(When calculating for high-speed roads, 0.8 m/s^3 is taken, and for ordinary roads, 1 m/s^3).

R is the radius of the curve, m.

Based on the parameters below, the correct transition curve is verified.

For roads in Azerbaijan, $J = 0.8\ m/s^3$ is accepted.

With values $R = 100\ m,\ V = 50\ km\ /\ h$ (in accordance with the normative table 1) from expression (9) we calculate l and find the values l = 33.2447 m. As can be seen from table 2, the length of the studied transition curve is 42 , 0988 m. And this fully corresponds to the conditions determined by expression (9).

The rate of increase of the centrifugal acceleration of a vehicle moving at a constant speed along the transition curve is determined as follows:

$$J(t) = k(t) \cdot \frac{V^3}{L(t)} \tag{10}$$

From expression (10) it can be seen that when the car crosses the transition curve, the rate of increase of centrifugal acceleration monotonously changes from 0 (at the beginning of the transition curve) to the end of the transition curve.

In expression (3), the curvature at the start point is 0, and at the end point $-1/r$. This provides a smooth connection of the circular path of the road with a straight line using a transition curve expressed by formula (3).

As can be seen from Figure 3, the curvature changes smoothly and monotonically along the transition curve.

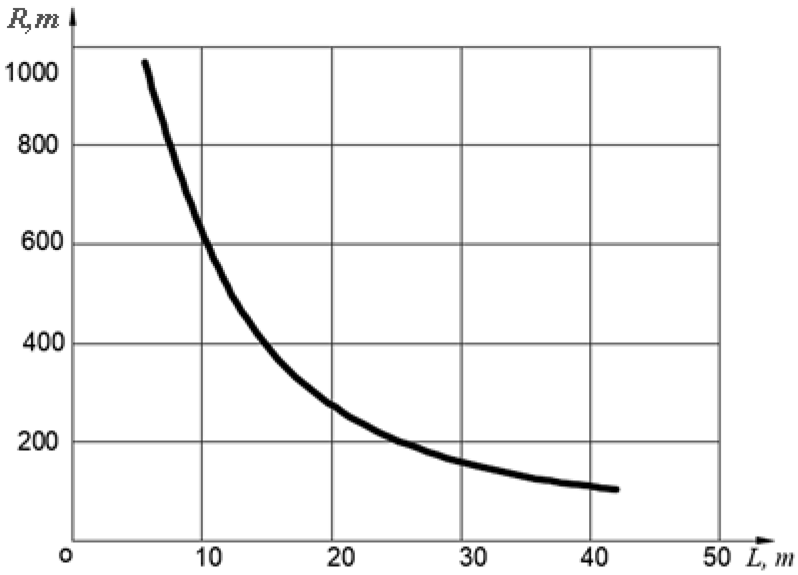


Figure 3. Graph of the dependence of the radius of curvature of the transition curve from the path traveled

Main conclusions. According to the proposed method, a monotonic change in the centrifugal acceleration along the transition curve, starting from zero (Figure 3), determines a smooth change in the centrifugal force. The application of this method allows more suitable roads to be designed and built with minimal braking when entering and exiting curves with increasing acceleration. In addition, the use of curves expressed by formula (3) on curved road sections more realistically and accurately determines the trajectory of the car when cornering.

REFERENCES

[1] Piriyev Y.M. Avtomobil yolları. Dərslik. Bakı, «Azərbaycan» nəşriyyatı, 1999. -560 s.

[2] А.И. Титов, И.И. Позняк, К.Ю. Соколова. Разбивочные работы. Методические указания к расчетно-графическим работам по инженерной геодезии. Москва, МАДИ, 2014. -29 с.

[3] Иманов А.С. Профилирование лопаток по геометрическому критерию качества на основе решения обратной задач. Авиационная техника: Изв. Вузов. – 2003г. - №1. –С. 64-66.

[4] А.С. Иманов, П.Ш. Абдуллаев. Профилирование плоских авиационных лопаток на базе дифференциального уравнения кривизны. Запорожье АО «Мотор Сич», Вестник двигателестроения, № 2. 2015 г. - С. 154-158.

[5] Халилов И.А., Иманов А.С. Моделирование кулачкового механизма с учетом критериев качества. Вестник Херсонского технического университета: 4(63), 2017 г., -с. 126-134.

[6] С.И. Булдаков. Особенности проектирования автомобильных дорог. Учебное пособие. Екатеринбург, 2016. -271 с.

[7] Халилов И.А., Керимов С.Х., Иманов А.С. Аналитическое определение радиуса кривизны профиля кулачка кулачкового механизма с роликовым толкателем / Вестник Азербайджан / Инженерной Академии / Баку: 2017, Т9, №4, 6 с.

[8] İmanov Ə.S., Xəlilov İ.Ə. İki təsirli lövhəli hidromaşınların statorlarının profilinin əyrilik əsasında qurulması. Azər. və Tür. Universitetləri: «Təhsil, Elm, Texnologiya» I beynəlxalq elm. Prak. konfr. III hissə. 2019, səh.5

[9] İmanov Ə.S., Xəlilov İ.Ə. Əyriliyin kefiyyət göstəricisi əsasında qurulmuş yumruqlu mexanizmin kinematik analizi. «Ölçmə və kefiyyət: problemlər, prespektivlər» mövzusunda Beynəlxalq Elmi-texniki konfrans materialları, Aztu, Bakı 2018-ci il. səh. 182-186.