On the Problem of Identification of the Hydraulic Resistance Coefficient of a Pipeline

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***Abstract*—We consider a problem of identification of the hydraulic resistance coefficient of a main pipeline's section when transporting hydrocarbon raw materials. The considered identification problem is reduced to a class of finite-dimensional optimization problems, for the solution of which we propose to use numerical finite-dimensional first-order optimization methods. For this purpose, we derive formulas for the components of the gradient of the objective functional in the space of identifiable parameters. The obtained values of the optimizable vector can then be used to construct the identifiable function from any class of basis functions using interpolation and approximation techniques. We also carried out some numerical experiments, the results of which are presented.**

Keywords—distributed system, hydraulic resistance coefficient, inverse problem, identification problem, adjoint problem, gradient of functional.

I. INTRODUCTION

In recent years, pipeline transportation systems have become the main and most significant means of transporting hydrocarbon raw materials. When a fluid moves through a pipeline, two sources of energy loss occur: losses along the length of the pipeline and losses in local resistances. Local resistances are due to the presence of all kinds of devices that have short length in comparison with the length of the pipe, in which there takes place a sharp change of velocity in magnitude and/or direction, as well as various locking devices, turns, valves, etc. Losses in local resistances result from increased fluid mixing, followed by vortex formation, and large velocity gradients. Losses along the length of the pipe or friction losses arise due to friction in the flow and depend on the length of the pipeline and on the hydraulic resistance coefficient (see [1,2]). Resistance values of pipeline sections have a significant impact on the movement of oil (gas) in the pipeline. It is known (see [3]) that the resistance of a pipeline section depends not only on the geometric dimensions of the pipe, such as the length of the section and the diameter of the pipe, but also on the roughness of the inner surface of the pipe, which affects the value of the hydraulic resistance coefficient. In the course of long-term operation of pipelines, the hydraulic resistance coefficients of the sections change, namely, they increase as a result of the formation of hydrates on the inner surface of the pipelines. These processes occur most intensively as a result of disruption of technological operation modes of pipelines at installations for the preparation of hydrocarbon raw materials transportation. The development of main pipeline networks for hydrocarbon raw materials transportation is accompanied by the wide and intensive use of modern measuring and computer equipment, as well as remote control systems. Therefore, it is of great practical importance to use mathematical methods for identifying hydraulic resistance coefficients for operational monitoring of the state of pipeline network sections that makes it possible to identify suspicious areas where hydrate formation processes have begun actively, which significantly affects the loss of oil (gas) energy, as a result of which it is necessary to use more intensively artificial increase in the energy of oil (gas) on pumping (compressor) stations. Accidents are also possible at pipeline sections due to hydration and formation of hydraulic jams in them. Currently, to determine the hydraulic resistance coefficient in a pipeline, there are a large number of empirical and semi-empirical formulas derived from experimental studies (see [4,5]). These formulas involve parameters, in particular, the roughness coefficient, the determination of the value of which poses certain difficulties. In this regard, it is important to conduct research to determine the hydraulic resistance coefficient individually for each section of a particular pipeline system. The problem of identification of the hydraulic resistance coefficient of a pipeline section considered in the paper belongs to the class of coefficient-inverse problems for systems with distributed parameters. Here we take into account the dependence of the identifiable function on the current state of the process, namely, on the fluid velocity, $\omega(x,t)$, and on the pipeline point, $x \in (0,\ell)$. The initial data for solving the identification problem are the known values of fluid pressure and/or fluid motion velocity, observed continuously or at discrete times at different points of the pipeline section. For numerical solution of the problem, we employ a projection approach, based on finding optimal zonal values of the parameters of the identifiable coefficient on a given class of functions, which is defined by means of known basis functions. To do this, the entire section is divided into a given number of subsections, for each of which the set of possible feasible values of the process operating modes is divided into a given number of phase subsets (zones). For each

such subset and subsection, the hydraulic resistance coefficient is sought in the form of a representation (depending on the operating mode) using basis functions. With respect to the ends of the subsections and representation parameters for each subsection, we have obtained formulas for the gradient of the objective functional, which make it possible to use efficient first-order optimization methods. The results of the carried out numerical experiments are presented using a few test problems.

II. Problem Statement

Consider non-stationary motion of an incompressible fluid with constant density, ρ, and viscosity, υ, along a linear horizontal pipeline section of length ℓ and diameter d. This process can be adequately described by the Charny-linearized system of hyperbolic type differential equations (see [6]):

$$-\frac{\partial p}{\partial x} = \rho\left[\frac{\partial \omega}{\partial t} + \alpha\,\lambda\,\omega\right], -\frac{\partial p}{\partial t} = c^2\rho\frac{\partial \omega}{\partial x}, \qquad (1)$$

$$x \in (0,\ell), \qquad t > t_0.$$

Here $p = p(x,t)$ and $\omega = \omega(x,t)$ define the fluid flow regime, namely the pressure and velocity of flow at a point $x \in (0,\ell)$ of the pipeline at point of time $t > t_0$; c is the speed of sound in the fluid; α is the linearization coefficient defined, generally speaking, by the average velocity of the fluid in the pipeline. It is known (see [7]) that the hydraulic resistance coefficient, λ, participating in (1) depends on the fluid motion regime (laminar or turbulent), i.e. on the Reynolds number, $\mathrm{Re} = \omega d/\upsilon$. The coefficient λ also depends on the relative roughness of the inner surface of the pipeline section, $\varepsilon = k/d$. Here k is the value of absolute roughness characterizing the state of the inner surface of the pipeline. The range of possible values of the roughness coefficient, k, is quite large; its values for new pipes depend on the manufacturing quality and material, and for old pipes they mainly depend on the period, operating conditions, and properties of the transported fluid. In all known formulas for the hydraulic resistance coefficient (see [3]), an essential role is played by the absolute roughness coefficient, k, of the pipe surface, direct measurement of the value of which is not possible in practice. On the other hand, studies related to the determination of the hydraulic resistance coefficient are important, given the significant influence of their values on the consumption of energy resources spent on the transportation of hydrocarbon raw materials. Knowing the value of the friction coefficient, on the one hand, allows monitoring the current state of pipelines, planning and carrying out appropriate preventive maintenance work to clean the inner surface of the pipeline in time. On the other hand, control of the fluid pumping-over process under non-stationary flow regimes arising from planned and unplanned transient processes, as well as the calculation of optimal operating modes of technological equipment is carried out using a mathematical model of the process (see [8]). Equation (1) and the hydraulic resistance coefficient of the pipeline section participating in (1) play an important role in calculations. Taking all of the above into account, the paper proposes a formulation and numerical solution of the problem of determining the hydraulic resistance coefficient of a pipeline section under specific operating conditions and the current state of its inner surface. Considering that the roughness is different for different sections of the pipeline, we conclude that λ is a function of the velocity, ω, and the point of the pipe, x, i.e., $\lambda = \lambda(\omega, x)$. At that, the value of the roughness coefficient of the inner surface for a given section is taken into account by the dependence $\lambda(\omega, x)$ itself.

Let there be the results of any two of the following three types of pairs of observations of pumping-over modes at pumping stations located at the ends of the section, carried out in N different time intervals $[\underline{T}_i, \overline{T}_i]$, $i = 1,2,\dots,N$:

$$p_i(0,t) = \varphi_{0i}(t), \qquad p_i(\ell,t) = \varphi_{\ell i}(t), \qquad (2)$$

$$p_i(0,t) = \varphi_{0i}(t), \qquad \omega_i(\ell,t) = \psi_{\ell i}(t), \qquad (3)$$

$$\omega_i(0,t) = \psi_{0i}(t), \qquad p_i(\ell,t) = \varphi_{\ell i}(t), \qquad (4)$$

assuming a sufficiently long duration of each of the time intervals $[\underline{T}_i, \overline{T}_i]$. The observable functions $\varphi_{0i}(t)$, $\varphi_{\ell i}(t)$, $\psi_{0i}(t)$, $\psi_{\ell i}(t)$ are assumed to be continuous on $[\underline{T}_i, \overline{T}_i]$, $i = 1,2,\dots,N$. For practical reasons, if it is impossible to measure the operating mode simultaneously at all points of the pipeline, we will assume that only the operating modes at the ends of the pipeline are observable, and the state of the initial operating modes for each of the observable processes at $t \in [\underline{T}_i, \overline{T}_i]$ is not specified. Therefore, we will consider the processes described by (1) without initial conditions. The research of such problems was initiated by A.N. Tikhonov (see [9]). In [10], they carried on studies of the duration of the influence of initial conditions on evolutionary processes and it was discovered that this duration is finite within practical accuracy and mainly determined by the viscosity of the fluid and the geometric dimensions of the pipeline section, but does not depend on the initial and boundary conditions themselves. Note that to determine the duration of the influence of the initial conditions for each specific section, taking the properties of the pumped fluid into account, it is necessary to carry out special calculations recommended in [10] only once. Let's denote the duration of this influence by ΔT; we assume that the duration of the observation intervals $[\underline{T}_i, \overline{T}_i]$, $i = 1,2,\dots,N$, much exceed the duration ΔT. The solution of differential equations with unspecified initial conditions and given boundary conditions is usually called the boundary mode. To calculate the boundary mode for (1) in the case of a given hydraulic resistance coefficient, some boundary conditions must be specified. In the case of specifying one of the types of conditions (2) – (4), the problem of calculating boundary modes is correct. For a direct numerical solution of the problem and obtaining the boundary mode on the time interval $[\underline{T}_i + \Delta T, \overline{T}_i]$, for example, using the grid method for each i-th observation experiment, it is necessary to set some initial conditions at $t = \underline{T}_i$. For this purpose, considering the process steady up to the moment of time $\underline{T}_i$, $i = 1,2,..,N$, i.e. $\omega(x,t) = \mathrm{const}$, $x \in [0,\ell]$, $t \le \underline{T}_i$, integrating the first equation in (1) over x and using, for example, the available values of the boundary conditions for pressure and velocity at the left or right ends at $t = \underline{T}_i$, the following values can be taken as initial conditions:

$$p_i(x,\underline{T}_i) = \left[\varphi_{\ell i}(\underline{T}_i) - \varphi_{0i}(\underline{T}_i)\right]\frac{x}{\ell} + \varphi_{0i}(\underline{T}_i),$$

$$\omega_i(x,\underline{T}_i) = \frac{\left[\varphi_{0i}(\underline{T}_i) - \varphi_{\ell i}(\underline{T}_i)\right]}{\beta\cdot\ell}, x \in [0,\ell], \qquad (5)$$

in the case of specifying the boundary conditions as (2) (here β is some approximate value for the product $\alpha\cdot\lambda$);

$$p_i(x,\underline{T}_i) = \varphi_{0i}(\underline{T}_i) - \beta\cdot\psi_{\ell i}(\underline{T}_i)\cdot x,$$

$$\omega_i(x,\underline{T}_i)=\psi_{\ell i}(\underline{T}_i),\qquad x\in[0,\ell], \tag{6}$$

in the case of specifying the boundary conditions as (3);

$$p_i(x,\underline{T}_i)=\varphi_{\ell i}(\underline{T}_i)+\beta\cdot\psi_{0i}(\underline{T}_i)\cdot(\ell-x),$$
$$\omega_i(x,\underline{T}_i)=\psi_{0i}(\underline{T}_i),\qquad x\in[0,\ell], \tag{7}$$

in the case of specifying the boundary conditions as (4).

The unknown hydraulic resistance coefficient, $\lambda(\omega,x)$, can be defined by distinct formulas at different speed ranges and have discontinuities of the first kind. Therefore, the solution of the considered boundary-value problems, i.e., the pair $\big(p(x,t),\omega(x,t)\big)$ will be assumed to be continuous in $x\in(0,\ell)$ and differentiable in $t\in[\underline{T}_i,\overline{T}_i]$.

In order to determine the required hydraulic resistance coefficient, $\lambda(\omega,x)$, for possible types of observation (2) – (4), we introduce the following functionals of standard deviation of

1) the observable boundary conditions

$$\omega_i(0,t)=\psi_{0i}(t),\qquad \omega_i(\ell,t)=\psi_{\ell i}(t) \tag{8}$$

from those calculated as a result of solving the problem (1), (2), and (5):

$$\mathrm{F}(\lambda)=\frac{1}{N}\sum_{i=1}^{N}\mathrm{F}_i(\lambda;\varphi_{0i},\varphi_{\ell i})+\varepsilon\|\lambda-\hat{\lambda}\|^2, \tag{9}$$

$$\mathrm{F}_i(\lambda;\varphi_{0i},\varphi_{\ell i})=\int_{\underline{T}_i+\Delta T}^{\overline{T}_i}\{[\omega(0,t;\lambda;\varphi_{0i},\varphi_{\ell i})-\psi_{0i}(t)]^2+$$
$$+[\omega(\ell,t;\lambda;\varphi_{0i},\varphi_{\ell i})-\psi_{\ell i}(t)]^2\}\mathrm{d}t,$$

where $\omega(x,t;\lambda;\varphi_{0i},\varphi_{\ell i})$, $x\in(0,\ell)$, $t\in[\underline{T}_i+\Delta T,\overline{T}_i]$, $i=1,2,\dots,N$, is the solution of problem (1), (2), and (5);

2) the observable boundary conditions (4) from those calculated as a result of solving the problem (1), (3), and (6):

$$\mathrm{F}(\lambda)=\frac{1}{N}\sum_{i=1}^{N}\mathrm{F}_i(\lambda;\varphi_{0i},\psi_{\ell i})+\varepsilon\|\lambda-\hat{\lambda}\|^2, \tag{10}$$

$$\mathrm{F}_i(\lambda;\varphi_{0i},\psi_{\ell i})=\int_{\underline{T}_i+\Delta T}^{\overline{T}_i}\{[\omega(0,t;\lambda;\varphi_{0i},\psi_{\ell i})-\psi_{0i}(t)]^2+$$
$$+[p(\ell,t;\lambda;\varphi_{0i},\psi_{\ell i})-\varphi_{\ell i}(t)]^2\}\mathrm{d}t,$$

where $p(x,t;\lambda;\varphi_{0i},\psi_{\ell i})$ and $\omega(x,t;\lambda;\varphi_{0i},\psi_{\ell i})$, $x\in(0,\ell)$, $t\in[\underline{T}_i+\Delta T,\overline{T}_i]$, $i=1,2,\dots,N$, is the solution of problem (1), (2), and (6);

3) the observable boundary conditions (3) from those calculated as a result of solving the problem (1), (4), and (7):

$$\mathrm{F}(\lambda)=\frac{1}{N}\sum_{i=1}^{N}\mathrm{F}_i(\lambda;\psi_{0i},\varphi_{\ell i})+\varepsilon\|\lambda-\hat{\lambda}\|^2, \tag{11}$$

$$\mathrm{F}_i(\lambda;\psi_{0i},\varphi_{\ell i})=\int_{\underline{T}_i+\Delta T}^{\overline{T}_i}\{[p(0,t;\lambda;\psi_{0i},\varphi_{\ell i})-\varphi_{0i}(t)]^2+$$
$$+[\omega(\ell,t;\lambda;\psi_{0i},\varphi_{\ell i})-\psi_{\ell i}(t)]^2\}\mathrm{d}t,$$

where $p(x,t;\lambda;\psi_{0i},\varphi_{\ell i})$ and $\omega(x,t;\lambda;\psi_{0i},\varphi_{\ell i})$, $x\in(0,\ell)$, $t\in[\underline{T}_i+\Delta T,\overline{T}_i]$, $i=1,2,\dots,N$, is the solution of problem (1), (2), and (7). The second terms in the considered functionals (9) – (11) are for regularization purposes, where ε and $\hat{\lambda}$ are regularization parameters (see [11]).

Thus, the considered problem of determining the hydraulic resistance coefficient of a pipeline section has been reduced to the problem of minimizing one of the functionals (9), (10), or (11) taking into account the mathematical model of the process, (1), and one of the boundary conditions, (2), (3), or (4). In this case, the initial conditions can be chosen from (5) – (7) rather arbitrarily, since they do not affect the value of the functional. The present problem is to be called the inverse problem or the parametric identification problem with unspecified initial conditions.

In the general case, it is possible that observations of pumping-over modes are carried out not only at the ends of the section, but also at some intermediate points $x_j\in(0,\ell)$, $j=1,2,\dots,M$. Most often, in practice, it is technically possible to measure the current pressure values at intermediate points of the pipeline section, i.e.

$$p(x_j,t)=\varphi_{ji}(t),t\in[\underline{T}_i,\overline{T}_i],j=1,2,\dots,M,i=1,2,\dots,N.$$

In this case, instead of functionals (9) – (11), one should consider the functional

$$\hat{\mathrm{F}}(\lambda)=\mathrm{F}(\lambda)+\frac{1}{N\cdot M}\sum_{i=1}^{N}\sum_{j=1}^{M}\int_{\underline{T}_i+\Delta T}^{\overline{T}_i}\big[p(x_j,t)-\varphi_{ji}(t)\big]^2\mathrm{d}t.$$

It is clear that when solving the inverse problem, an increase in the number of pipeline points at which measurements are made and in the duration of measurements leads to an increase in the accuracy of the results, taking into account the inevitable errors and interference in measurements.

III. Numerical Solution of the Problem

The considered problem of determining the hydraulic resistance coefficient function is formulated within the framework of the optimal control problem for a system with distributed parameters, and more precisely, the feedback control problem, since the current value of the function to be identified for all $x\in(0,\ell)$ and $t\in[\underline{T}_i,\overline{T}_i]$, $i=1,2,\dots,N$, depends on the current state of the process $\omega(x,t)$ and the point of the pipeline $x\in(0,\ell)$. For numerical solution of the problem, we propose to use an approach based on the idea outlined in [12, 13]. We split the entire segment $X=[0,\ell]$ by intermediate ordered (subsequently optimized) (L_x-1) points ξ_k, $k=0,1,\dots,L_x$, such that $0=\xi_0<\xi_1<\dots<\xi_{L_x-1}<\xi_{L_x}=\ell$, into L_x subsections $X_k=[\xi_{k-1},\xi_k)$. Suppose that, based on a priori information on the process, the range of actually possible values of the fluid motion velocity in the pipeline is known, i.e.

$$\omega(x,t)\in\Omega=[\underline{\omega},\overline{\omega}],\qquad x\in(0,\ell),\qquad t>t_0,$$

where $\underline{\omega}$ and $\overline{\omega}$ are known limit values of the function $\omega(x,t)$. We quantize the set Ω with predetermined ordered values ω_j, $j=0,1,\dots,L_\omega$, such that $\underline{\omega}=\omega_0<\omega_1<\dots<\omega_{L_\omega}=\overline{\omega}$, and introduce subsets (zones) $\Omega_j=[\omega_{j-1},\omega_j)\subset\Omega$, $j=1,2,\dots,L_\omega$. In particular, quantization can be uniform or dependent on a priori information on the process.

The function to be identified, which determines the hydraulic resistance coefficient $\lambda(\omega, x)$, will be determined for each zone Ω_j and for each subsection X_k by means of predetermined L basis functions, namely, the function $\lambda(\omega, x)$ is sought in the form

$$\lambda(\omega, x) = \sum_{s=1}^{L} \lambda_s^{kj} \cdot \gamma_s(\omega), \qquad \omega \in \Omega_j, \qquad x \in X_k \quad (12)$$

Here λ_s^{kj} are unknown coefficients to be identified, and $\gamma_s(\omega)$, $s = 1, 2, \dots, L$, is a set of known linearly independent basis functions. Thus, the problem of finding the function $\lambda(\omega, x)$ has been reduced to the problem of finding finite-dimensional vectors $\Lambda = \left(\lambda_s^{kj}\right)$, $j = 1,2, \dots, L_\omega$, $k = 1,2, \dots, L_x$, $s = 1,2, \dots, L$, and $\xi = \left(\xi_1, \xi_2, \dots, \xi_{L_x}\right)$. We denote by $z = (\Lambda, \xi)$ a vector of dimension $L \cdot L_x \cdot (L_\omega + 1) + L_x - 1$. The original optimal control problem has been reduced to the parametric optimal control problem to find a finite-dimensional vector z. To solve it, one can use numerical methods based on the construction of a minimizing sequence $\{z^k\}$ using iterative first-order optimization procedures, for example, the conjugate gradients method (see [14]). To construct a procedure for first-order numerical optimization, we need formulas for the components of the gradient of the objective functional; to obtain them, we use the well-known technique of variation of the optimizable parameters of the functional (see [13,15,16]) (the derivation of these formulas is omitted). The components of the gradient of the objective functional of the considered problems on the class of functions (12) with respect to unknown parameters λ_s^{kj} are determined by the formulas

$$\frac{\partial F(z)}{\partial \lambda_s^{kj}} = \frac{1}{N} \sum_{i=1}^{N} \int_{\underline{T}_i + \Delta T}^{\overline{T}_i} \int_{\Pi_{kj}^i(t;z)} \alpha\rho q(x,t) \gamma_s(\omega) \omega(x,t,z)\, dx\, dt$$

$$s = 1,2, \dots, L; \quad j = 1,2, \dots, L_\omega; \quad k = 1,2, \dots, L_x,$$

and with respect to the unknown parameters $\xi_1, \xi_2, \dots, \xi_{L_x}$ by the formulas

$$\frac{\partial F}{\partial \xi_k} = \frac{1}{N} \sum_{i=1}^{N} \sum_{s=1}^{L} \int_{\underline{T}_i + \Delta T}^{\overline{T}_i} \alpha\rho \left[\lambda_s^{k-1,j} - \lambda_s^{k,j}\right] \gamma_s(\omega) q(x,t)$$

$$\omega(x,t;z)\mathrm{d}t, \quad k = 1,2, \dots, L_x - 1.$$

Here the set $\Pi_{kj}^i(t;z)$ for each moment in time $t \in \left[\underline{T}_i + \Delta T, \overline{T}_i\right]$ is defined as the union of those intervals of the sub-segment X_k, in which, under the i-th observation of the process at moment of time t, the velocity $\omega(x,t;z) \in \Omega_j$. The pair $(q(x,t), v(x,t))$ is a solution to the system of equations conjugate to (1):

$$\frac{\partial q(x,t)}{\partial x} + \frac{\partial v(x,t)}{\partial t} = 0,$$

$$\rho \frac{\partial q(x,t)}{\partial t} + c^2 \rho \frac{\partial v(x,t)}{\partial x} - \alpha\rho q(x,t) \left[\lambda + \omega(x,t) \frac{d\lambda}{d\omega}\right] = 0$$

for which the boundary and initial conditions, depending on the problem under consideration, have different forms:

a) for the problem (1), (3), (6), and (10):

$$q(\ell, t) = -2[p(\ell, t) - \varphi_{\ell i}(t)],$$

$$v(0,t) = \frac{2}{c^2 \rho}[\omega(0,t) - \psi_{0i}(t)],$$

$$q\left(x, \overline{T}_i\right) = 0, \qquad v\left(x, \overline{T}_i\right) = 0,$$

$$x \in (0, \ell), \qquad t \in \left[\underline{T}_i + \Delta T, \overline{T}_i\right];$$

b) for the problem (1), (4), (7), and (11):

$$q(0,t) = 2[p(0,t) - \varphi_{0i}(t)],$$

$$v(\ell, t) = -\frac{2}{c^2 \rho}[\omega(\ell, t) - \psi_{\ell i}(t)],$$

$$q\left(x, \overline{T}_i\right) = 0, \qquad v\left(x, \overline{T}_i\right) = 0,$$

$$x \in (0, \ell), \qquad t \in \left[\underline{T}_i + \Delta T, \overline{T}_i\right];$$

c) for the problem (1), (2), (5), and (9):

$$v(0,t) = \frac{2}{c^2 \rho}[\omega(0,t) - \psi_{0i}(t)],$$

$$v(\ell, t) = -\frac{2}{c^2 \rho}[\omega(\ell, t) - \psi_{\ell i}(t)],$$

$$q\left(x, \overline{T}_i\right) = 0, \qquad v\left(x, \overline{T}_i\right) = 0,$$

$$x \in (0, \ell), \qquad t \in \left[\underline{T}_i + \Delta T, \overline{T}_i\right].$$

An important role in identifying the unknown function $\lambda(\omega, x)$ using the described approach is played by the choice of both the zones Ω_j, $j = 1,2, \dots, L_\omega$, and their number, as well as the number of points ξ_k, $k = 1,2, \dots, L_x - 1$. The following approach is recommended. Firstly, the initial values of L_x and L_ω are arbitrarily chosen, the boundaries of the subsections ξ_k, $k = 1,2, \dots, L_x - 1$ are specified, and some zones Ω_j, $j = 1,2, \dots, L_\omega$, are chosen. Having solved the posed identification problem, the obtained values of the identifiable parameters are analyzed for all neighboring subsections and zones. If the identifiable parameters in any two adjacent subsections or zones differ by a sufficiently small amount, then these adjacent subsections or zones can be combined into one, thus decreasing the numbers L_x and/or L_ω. If the identifiable parameters in two adjacent subsections or zones differ significantly, then, on the contrary, each of these adjacent subsections or zones should be divided, for example, into two parts, thus increasing the numbers L_x and/or L_ω, and again solve the posed identification problem. An increase in the number of subsections and zones should be carried out until the value of the objective functional ceases to change (decrease) significantly.

IV. Result of Numerical Experiments

Numerous computational experiments have been carried out on test (model) problems. Here are some of the results of these experiments. Let us assume that we are observing non-stationary oil transportation processes with density $\rho = 860\ \mathrm{kg/m^3}$ on a straight pipeline section with diameter $d = 0.53$ m and length $\ell = 100$ km. We take the speed of sound in oil equal to $c = 1200$ m/s, the value of the linearization parameter to be $\alpha = 1$. Suppose that we have the following results of $N = 10$ observations of the form (2) over the pumping-over modes (pressure) on pumping stations located at the ends of the segment; observations have been performed over ten different time segments $[\underline{T}_i, \overline{T}_i]$, $i = 1,2, \dots, 10$, each

of duration 120 min., i.e., $\overline{T}_i = \underline{T}_i + 120$, and, moreover, $\underline{T}_1 = 0$, $\underline{T}_i = \underline{T}_{i-1} + 180$, for $i = 2,3,\dots,10$:

$$p_1(0,t) = \left[5\cos\left(0.0009\left(t-\underline{T}_1\right)\right)+20\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_1(\ell,t) = \left[2\sin\left(0.0009\left(t-\underline{T}_1\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_1, \overline{T}_1\right]; \qquad (13)$$

$$p_2(0,t) = \left[6\cos\left(0.0009\left(t-\underline{T}_2\right)\right)+30\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_2(\ell,t) = \left[2\sin\left(0.0008\left(t-\underline{T}_2\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_2, \overline{T}_2\right]; \qquad (14)$$

$$p_3(0,t) = \left[7\cos\left(0.0009\left(t-\underline{T}_3\right)\right)+40\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_3(\ell,t) = \left[2\sin\left(0.0009\left(t-\underline{T}_3\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_3, \overline{T}_3\right]; \qquad (15)$$

$$p_4(0,t) = \left[8\cos\left(0.0009\left(t-\underline{T}_4\right)\right)+50\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_4(\ell,t) = \left[3\sin\left(0.0007\left(t-\underline{T}_4\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_4, \overline{T}_4\right]; \qquad (16)$$

$$p_5(0,t) = \left[9\cos\left(0.0009\left(t-\underline{T}_5\right)\right)+60\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_5(\ell,t) = \left[4\sin\left(0.0006\left(t-\underline{T}_5\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_5, \overline{T}_5\right]; \qquad (17)$$

$$p_6(0,t) = \left[10\cos\left(0.0009\left(t-\underline{T}_6\right)\right)+70\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_6(\ell,t) = \left[5\cos\left(0.0007\left(t-\underline{T}_6\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_6, \overline{T}_6\right]; \qquad (18)$$

$$p_7(0,t) = \left[10\cos\left(0.0009\left(t-\underline{T}_7\right)\right)+80\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_7(\ell,t) = \left[4\cos\left(0.0008\left(t-\underline{T}_7\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_7, \overline{T}_7\right]; \qquad (19)$$

$$p_8(0,t) = \left[9\cos\left(0.0009\left(t-\underline{T}_8\right)\right)+90\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_8(\ell,t) = \left[3\sin\left(0.0009\left(t-\underline{T}_8\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_8, \overline{T}_8\right]; \qquad (20)$$

$$p_9(0,t) = \left[8\cos\left(0.0009\left(t-\underline{T}_9\right)\right)+100\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_9(\ell,t) = \left[2\sin\left(0.0007\left(t-\underline{T}_9\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_9, \overline{T}_9\right]; \qquad (21)$$

$$p_{10}(0,t) = \left[10\cos\left(0.001\left(t-\underline{T}_{10}\right)\right)+110\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$p_{10}(\ell,t) = \left[5\sin\left(0.001\left(t-\underline{T}_{10}\right)\right)+10\right]98066.5\,\frac{\text{kg}}{\text{m}^3},$$

$$t \in \left[\underline{T}_{10}, \overline{T}_{10}\right]. \qquad (22)$$

To obtain some test observable values of raw material consumption at the ends of the pipeline section, the section X is divided by one ($L_x = 1$) point $\xi = 70$ km into 2 subsections, $X_1 = [0,70]$ and $X_2 = [70,100]$. From preliminary numerical calculations for various permissible values of hydraulic resistances, the range of possible speeds in the entire section was identified to be $\underline{\omega} = 0.35$ m/s and $\overline{\omega} = 2.65$ m/s. That is why the following 10 zones, Ω_i, $i = 1,2,\dots,10$, were considered for Ω:

$$\{[0.35;0.5),[0.5;0.75),[0.75;1),[1;1.25),[1.25;1.5),$$
$$[1.5;1.75),[1.75;2),[2;2.25),[2.25;2.5),[2.5;2.65)\}.$$

For the test problems, we obtained the observable values of raw materials consumption at the ends of the section of the form (8) for ten observations by means of calculation. For this purpose, ten initial-boundary value problems were solved for (1) with the corresponding ten variants of boundary conditions (24) – (33) of the form (2) with the initial conditions calculated according to (5). Here are the results of solving two test problems. In the first test problem, to obtain the observable values of the form (8), the hydraulic resistance coefficients for each zone and for all subsections were taken as linear functions

$$\lambda(\omega,x) = \lambda_1^{k,j} + \lambda_2^{k,j}\omega, \qquad k = 1,2, \qquad i = 1,2,\dots,10$$

i.e. the basis functions in (12) are $\gamma_1(\omega) = 1$, $\gamma_2(\omega) = \omega$, $L = 2$. The used (optimal) values of the parameters $\lambda_s^{k,j}$ are given in Table 1. The same class of functions was used to solve the problem of identifying the hydraulic resistance function. For the second test problem, the observable values (8) were obtained for the class of piecewise constant hydraulic resistance functions:

$$\lambda(\omega,x) = \lambda_1^{k,j}, \qquad k = 1,2, \qquad i = 1,2,\dots,10,$$

i.e. $L = 1$ and $\gamma_1(\omega) = 1$. The used (optimal) values of the parameters $\lambda_1^{k,j}$ are given in Table 3. To analyze the stability of the obtained solutions of the considered parametric identification problem, noises were introduced into data given by (8). Namely, we solved the inverse problems under conditions for which the observed values of the flow rate at the ends of the section (the results of solving the above direct boundary-value problems under the boundary conditions (13) – (22)) are subject to disturbances that have a uniform distribution on the interval $[-\chi,\chi]$. The value of $\chi \geq 0$ in the experiments varied and took the values 0 (i.e., without any noise), 0.05, and 0.1, which corresponded to noise level equal to 0%, 0.5%, and 1%, as if from the measured flow rates $\widehat{\omega}_i(0,t)$ and $\widehat{\omega}_i(\ell,t)$ at the ends of the segment:

$$\widehat{\omega}_i(0,t) = (1+\gamma_1)\omega_i(0,t), \widehat{\omega}_i(\ell,t) = (1+\gamma_2)\omega_i(\ell,t),$$
$$i = 1,2,\dots,N.$$

To generate random variables γ_1 and γ_2 uniformly distributed over the interval $[-\chi,\chi]$, we used a standard program for generating pseudorandom numbers.

To determine the value of ΔT, special calculations were carried out, namely, the boundary-value problem (1) and (13) – (22) was solved for the considered section of the pipeline under various initial conditions. As a result, the duration ΔT was determined to be about 10 minutes.

The numerical solution of the boundary-value problem (1) and (13) – (22) was carried out using the implicit scheme of the grid method with the steps in the variables x and t equal to $h_x = 100$ m and $h_t = 7.2$ s, respectively. To solve the optimization problem, the conjugate gradient method was used with the following optimization parameters: the accuracy of solving the auxiliary one-dimensional minimization problem is 10^{-4},the accuracy of solving the multidimensional minimization problem is 10^{-3}. In all test problems, the zero vector was taken as the initial values of the identifiable parameters, i.e. $\lambda_s^{k,j} = 0$, $\xi_1^0 = 6$. For example, the value of the objective functional in the first test problem at the initial point $z_0 = (\lambda^0, \xi^0)$ under the noise level of 0% was $F^0 \approx 1198267791$.

As can be seen from the results of numerical experiments given in Tables 1 – 4, the influence of a measurement error of up to 1% (for instance, when measuring pressure in practice, the error range of 0.5 – 1% is quite acceptable) on the obtained values of the identifiable parameters is insignificant; therefore, the graphical presentation of the results of the influence of measurement errors is not given due to low information content.

TABLE 1. OPTIMAL AND OBTAINED VALUES OF THE PARAMETERS OF THE HYDRAULIC RESISTANCE FUNCTIONS UNDER 0%, 0.5% AND 1% NOISE LEVELS FOR TEST PROBLEM 1.

Zone index (j)		Segment 1: $(\lambda_1^{1j}; \lambda_2^{1j})$	Segment 2: $(\lambda_1^{2j}; \lambda_2^{2j})$
1	$(\lambda_s^{kj})^*$	(0.0368; –0.0181)	(0.0404; –0.0188)
	0 %	(0.0330; –0.0198)	(0.0383; –0.0171)
	0.5 %	(0.03299; –0.01991)	(0.038173; –0.01721)
	1 %	(0.03254; –0.02074)	(0.03764; –0.01760)
2	$(\lambda_s^{kj})^*$	(0.0328; –0.0101)	(0.0361; –0.0102)
	0 %	(0.0329; –0.0089)	(0.0362; –0.0133)
	0.5 %	(0.03319; –0.00894)	(0.036043; –0.01340)
	1 %	(0.03391; –0.00914)	(0.03772; –0.01363)
3	$(\lambda_s^{kj})^*$	(0.0300; –0.0065)	(0.0332; –0.0063)
	0 %	(0.0307; –0.0062)	(0.0317; –0.0074)
	0.5 %	(0.03093; –0.00624)	(0.03195; –0.00738)
	1 %	(0.03042; –0.00617)	(0.03159; –0.00773)
4	$(\lambda_s^{kj})^*$	(0.0282; –0.0046)	(0.0313; –0.0044)
	0 %	(0.0318; –0.0068)	(0.0331; –0.0037)
	0.5 %	(0.03206; –0.00677)	(0.03296; –0.00369)
	1 %	(0.03138; –0.00711)	(0.03471; –0.00367)
5	$(\lambda_s^{kj})^*$	(0.0268; –0.0035)	(0.0299; –0.0033)
	0 %	(0.0286; –0.0042)	(0.0264; –0.0020)
	0.5 %	(0.02880; –0.00423)	(0.026558; –0.00199)
	1 %	(0.02824; –0.00416)	(0.02636; –0.00195)
6	$(\lambda_s^{kj})^*$	(0.0258; –0.0028)	(0.0288; –0.0026)
	0 %	(0.0298; –0.0050)	(0.0269; –0.0019)
	0.5 %	(0.02973; –0.00499)	(0.026859; –0.00189)
	1 %	(0.02922; –0.00515)	(0.02689; –0.00188)
7	$(\lambda_s^{kj})^*$	(0.0249; –0.0023)	(0.0279; –0.0021)
	0 %	(0.0264; –0.0030)	(0.0257; –0.0021)
	0.5 %	(0.02632; –0.00301)	(0.025650; –0.00211)
	1 %	(0.02738; –0.00299)	(0.02641; –0.00219)
8	$(\lambda_s^{kj})^*$	(0.0242; –0.0020)	(0.0272; –0.0017)
	0 %	(0.0249; –0.0023)	(0.0261; –0.0013)
	0.5 %	(0.02483; –0.00229)	(0.026057; –0.00131)
	1 %	(0.02563; –0.00228)	(0.02688; –0.00127)
9	$(\lambda_s^{kj})^*$	(0.0235; –0.0017)	(0.0266; –0.0014)
	0 %	(0.0259; –0.0027)	(0.0257; –0.0012)
	0.5 %	(0.02587; –0.00269)	(0.025925; –0.00119)
	1 %	(0.02660; –0.00267)	(0.025289; –0.00125)
10	$(\lambda_s^{kj})^*$	(0.0230; –0.0015)	(0.0261; –0.0012)
	0 %	(0.0201; –0.0004)	(0.0253; –0.0008)
	0.5 %	(0.02001; –0.00039)	(0.025495; –0.00079)
	1 %	(0.02110; –0.00039)	(0.026528; –0.00083)

TABLE 2. OBTAINED VALUES OF THE ENDS OF THE SUBSECTIONS AND OF THE FUNCTIONAL UNDER 0%, 0.5% AND 1% NOISE LEVELS FOR TEST PROBLEM 1.

	Noise 0 %	Noise 0.5 %	Noise 1 %
End of segment ξ	69.9999	70.3813	69.5986
Functional $J(z)$	0.011035	0.014134	0.025079

TABLE 3. OPTIMAL AND OBTAINED VALUES OF THE PARAMETERS OF THE HYDRAULIC RESISTANCE FUNCTIONS UNDER 0%, 0.5% AND 1% NOISE LEVELS FOR TEST PROBLEM 2.

Zone index j		Segment 1: λ_1^{1j}	Segment 2: λ_1^{2j}
1	$(\lambda_s^{kj})^*$	0.0295	0.0329
	0 %	0.02848	0.03519
	0.5 %	0.02903	0.03389
	1 %	0.02948	0.03657
2	$(\lambda_s^{kj})^*$	0.0265	0.0297
	0 %	0.02556	0.03186
	0.5 %	0.02519	0.03091
	1 %	0.02630	0.03282
3	$(\lambda_s^{kj})^*$	0.0244	0.0277
	0 %	0.02360	0.02953
	0.5 %	0.02383	0.02890
	1 %	0.02409	0.03043
4	$(\lambda_s^{kj})^*$	0.0230	0.0263
	0 %	0.02233	0.02782
	0.5 %	0.02256	0.02799
	1 %	0.02293	0.02854
5	$(\lambda_s^{kj})^*$	0.0220	0.0254
	0 %	0.02141	0.02675
	0.5 %	0.02162	0.02694
	1 %	0.02198	0.02771
6	$(\lambda_s^{kj})^*$	0.0212	0.0246
	0 %	0.02066	0.02583
	0.5 %	0.02086	0.02609
	1 %	0.02033	0.02644
7	$(\lambda_s^{kj})^*$	0.0206	0.0241
	0 %	0.02009	0.02526
	0.5 %	0.02029	0.02551
	1 %	0.02003	0.02581
8	$(\lambda_s^{kj})^*$	0.0200	0.0236
	0 %	0.01951	0.02469
	0.5 %	0.01971	0.0249
	1 %	0.01999	0.02563
9	$(\lambda_s^{kj})^*$	0.0196	0.0232
	0 %	0.01913	0.02425
	0.5 %	0.01933	0.02442
	1 %	0.01882	0.02381
10	$(\lambda_s^{kj})^*$	0.0192	0.0229
	0 %	0.01876	0.02389
	0.5 %	0.01895	0.02412
	1 %	0.01845	0.02365

TABLE 4. THE OBTAINED VALUES OF THE ENDS OF THE SUBSECTIONS AND OF THE FUNCTIONAL UNDER 0%, 0.5% AND 1% NOISE LEVELS FOR TEST PROBLEM 2.

	Noise 0 %	Noise 0.5 %	Noise 1 %
End of segment ξ	69.90	69.81	70.63
Functional $J(z)$	0.007674	0.009021	0.010147

V. CONCLUSION

In the paper, we have considered the numerical solution of the parametric identification problem for a mathematical model of a dynamic object with distributed parameters by the example of solving the problem of identifying the hydraulic resistance coefficient under a non-stationary mode of fluid motion through pipelines. It is known that the hydraulic resistance coefficient is a function of the fluid velocity. Empirical formulas proposed by many engineers for estimating the value of the hydraulic resistance coefficient do not always give adequate results on specific pipelines due to the fact that they do not take into account the specific features of these pipelines. The theoretical and practical results obtained in the paper make it possible to use them in automatic control systems on pipelines. We proposed an approach to numerical solution of the considered problem that consists in the fact that the entire pipeline section is divided into a given number of subsections, the ends of which are optimizable, and we introduced the concept of a "zone" for the optimizable parameters of the identifiable function depending on the current state of the object. The set of all phase states of the object is divided into a finite number of subsets (zones), in each of which the function to be identified is assumed to be constant, linear, or having some other form of functional dependence. The results obtained can also be used in mathematical modeling and solving inverse problems for many technological processes and technical objects, the identifiable parameters of which are functions of the state of the process (object).

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