Localization of Leak Locations in a Complex Pipeline Network of Fluid Transportation

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***Abstract*—An inverse problem for a pipeline network of complex loopback structure is solved numerically. The problem is to determine the locations and amounts of leaks from unsteady f low characteristics measured at some pipeline points.**

***Keywords*—pipeline network, leak, optimization, inverse problem, unsteady f low.**

I. INTRODUCTION

The features of the problem include impulse functions involved in a system of hyperbolic differential equations, the absence of classical initial conditions, and boundary conditions specified as nonseparated relations between the states at the endpoints of adjacent pipeline segments. The problem is reduced to a parametric optimal control problem without initial conditions, but with nonseparated boundary conditions. The latter problem is solved by applying first-order optimization methods [1]. Results of numerical experiments are presented.

This paper differs from many other studies [2] in which leak locations and amounts were determined either in a steady f low regime in a pipeline of complex structure or in a transient f low regime in a pipeline consisting of a single linear segment [3-5]. In this study, we numerically solve an inverse problem [6] of determining leak locations and amounts in an unsteady f low in a pipeline network of complex (loopback) structure[7-9]. The problem is described by a system made up of numerous subsystems of two hyperbolic partial differential equations with impulse actions specified at possible leakage points on pipeline network segments.

Another feature of the problem is the assumption that, due to the long duration of the process under study, exact information on its initial state is not available at the time of monitoring and that the states of the process (which is distributed in space) cannot be quickly measured at all points [10]. Instead, there is information on a variety of possible initial states of the process and some state (regime) characteristics are measured at certain pipeline points starting at this time. One more feature of the problem is that its boundary conditions are specified as nonseparated relations (determined by physical laws) between the states at the endpoints of adjacent pipeline segments..

II. STATEMENT OF THE PROBLEM

To simplify presentation of numerical schemes and to be specific, let us consider the pipe network, containing 8 segments as shown in figure 1. Numbers in brackets identify the nodes (or junctions). The set of nodes we denote by I : $I=\{k_1,...,k_N\}$; where $k_i, i=\overline{1,N}$ are the nodes; $N=|I|$ is the numbers of nodes in the network. Two numbers in parentheses identify two-index numbers of segments. The flow in these segments goes from the first index to the second (for example, the flow in the segment (1,2)

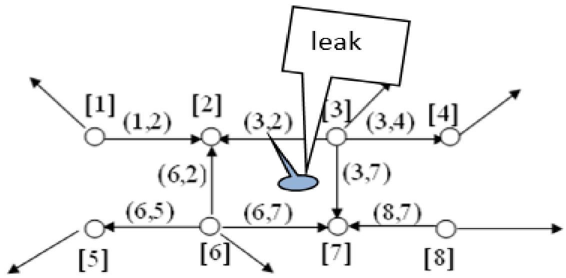

Fig.1 The scheme of pipe network with 8 nodes

is obviously from the node 1 to node 2.

Let J : $J=\{(k_i,k_j):k_i,k_j\in I\}$ is the set of segments and $M=|J|$ is it's quantity; $l_{k_ik_j}$, $d_{k_ik_j}$, $k_i,k_j\in I$ is a length and diameter of the segment (k_i,k_j) respectively; I_k^+ is the set of nodes connected with node k by segments where flow goes into the node, I_k^- is the set of nodes connected with node k by segments where flow goes out of the node; $I_k=I_k^+\bigcup I_k^-$ is the set of total nodes connected with node k and $N_k=|I_k|, N_{k^+}=|I_k^+|, N_{k^-}=|I_k^-|$, $N_k=N_{k^-}+N_{k^+}$.

Beside of inflows and outflows in the segments of the network there can be external inflows (sources) and outflows (sinks) with the rate $\tilde{q}_i(t)$ at some nodes $i\in I$ of the network. Positive and negative values of $\tilde{q}_i(t)$ indicate the existence of external inflow or outflow at the node i . However, in general case, assuming that the case $\tilde{q}_i(t)\equiv 0$ for the sources is admissible one can consider all nodes of the network as the nodes with external inflows or outflows. Let $I^f\subset I$ denote the set of nodes $i\in I$, where i is such that the set $I_i^+ \cup I_i^-$ consists of only one segment. It means that the node i is a node of external inflow or outflow for the whole pipe network (for example $I^f = \{1,4,5,8\}$ in fig.1). Let $N_f = |I^f|$ is the number of such nodes, it is obviously that $N_f \le N$; I^{int} is the set of nodes not belonging to I^f , so $N_{\text{int}} = |I^{\text{int}}|$, i.e., $I^{\text{int}} = I / I^f$, $N_{\text{int}} = N - N_f$. In actual conditions, the pumping stations are placed, the measuring equipment is installed and the quantitative accounting is conducted at the nodes from the set I^f .

We assume that at some instants of time $t \ge t_0$ at some points $\xi_{ks} \in (0,l)$, of any (k,s) -th section of the pipeline network, fluid leakage with the flow rates $q_{ks}^{loss}(t)$ began. Using the generalized Dirac function δ(x), we can describe the motion of the liquid by the following linearized system of differential equations for unsteady flow of dripping liquid with constant density ρ in a linear pipe (k,s) of length l_{ks} and diameter d_{ks} of oil pipeline network can be written in the following form [4-8]:

$$\begin{cases} -\dfrac{\partial P^{ks}(x,t)}{\partial x} = \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial t} + 2a^{ks} \dfrac{\rho}{S^{ks}} Q^{ks}(x,t), x \in (0,l^{ks}), \\ -\dfrac{\partial P^{ks}(x,t)}{\partial t} = c^2 \dfrac{\rho}{S^{ks}} \dfrac{\partial Q^{ks}(x,t)}{\partial x} + c^2 \dfrac{\rho}{S^{ks}} q_{ks}^{loss}(t)\delta(x-\xi_{ks}), \\ \qquad t \in (0,T],\ s \in I_k^+, k \in I. \end{cases} \quad (1)$$

here c is the sound velocity in the fluid; S^{ks} is the area of an internal cross-section of the segment (k,s) ; a^{ks} is the coefficient of dissipation (we may consider that the kinematic coefficient of viscosity γ is independent of pressure and the condition $2a^{ks} = \dfrac{32\gamma}{(d^{ks})^2} = const$ is quite accurate for a laminar flow). $Q^{k_i k_j}(x,t)$, $P^{k_i,k_j}(x,t)$ are the flow rate and pressure of flow, respectively, at the time instance t in the point $x \in (0, l^{k_i,k_j})$ of the segment (k_i,k_j) of the pipe network. $P^k(t), Q^k(t)$ are the pressure and flow rate at the node $k \in I$, respectively.

The conditions of Kirchhoff's first law (total flow into the node must be equal to total flow out of the node) are satisfied at the nodes of the network at $t \in [0,T]$:

$$\sum_{s \in I_k^+} Q^{ks}(l^{ks},t) - \sum_{s \in I_k^-} Q^{ks}(0,t) = \tilde{q}^k(t),\ k \in I \quad (3)$$

Also, the following conditions of flow continuity for the nodes of the net (the equality of the values of pressures on all adjacent ends of the segments of the network) hold:

$$P^k(t) = P^{k_i k}(l^{k_i k},t) = P^{kk_j}(0,t), k_i \in I_k^+, k_j \in I_k^-, k \in I, \quad (4)$$

where $\tilde{q}^k(t)$ is the external inflow $(\tilde{q}^k(t) > 0)$ or outflow $(\tilde{q}^k(t) < 0)$ for the node k , $P^k(t)$ is the value of the pressure in the node k . We must note that they have significant specific features, consisting in the fact that the conditions (3) and (4) are non-separated (nonlocal) boundary conditions unlike classical cases of boundary conditions for partial differential equations.

The total number of conditions for all nodes from I^f is N_f . So, the total number of conditions in (3) and (4) is $[N_f + N_{\text{int}}] + [(2M - N_f) - N_{\text{int}}] = 2M$. As it was noted above the number of conditions in (3) is N , but in view of the condition of material balance ($\sum_{k \in I} \tilde{q}^k(t) = 0$) for the whole pipeline network, we conclude that the number of linearly independent conditions is $N-1$. So, it is necessary to add any one independent condition. As a rule the value of pressure at one of the nodes $s \in I^f$ is given for this purpose, in place of the flow rate $q^s(t)$:

$$P^s(t) = \tilde{P}^s(t). \quad (5)$$

In the case of unknown points of leakages and their rates $\xi_{ks}, q_{ks}^{loss}(t)$ we will assume that at the ends of the pipeline sections a constant and rather long observation on pressure is made, i.e., the values of $P_{mes}^n(t),\ n \in I_p^f$ or $Q_{mes}^m(t), m \in I_q^f$ are known. It is quite natural to suppose that the sought leak spots do not coincide with the points of observation of regimes. In more general case, for every node from $I^f = I_q^f \cup I_p^f$, it is necessary to give the values of pressure ($I_p^f \subset I^f$ denotes the set of such nodes) or the values of flow rate (the set $I_q^f \subset I^f$) and I_p^f must not be an empty: $I_p^f \ne \varnothing$. So, we will add the following conditions to the condition (3):

$$\begin{cases} P^n(t) = P^{ns}(0,t) = P_{mes}^n(t), & s \in I_n^+,\ if\ I_n^- = \varnothing, \\ P^n(t) = P^{sn}(l^{sn},t) = P_{mes}^n(t), & s \in I_n^-,\ if\ I_n^+ = \varnothing, \end{cases} \quad n \in I_p^f, (6)$$

$$\begin{cases} Q^m(t) = Q^{ms}(0,t) = Q_{mes}^m(t), & s \in I_m^+,\ if\ I_m^- = \varnothing, \\ Q^m(t) = Q^{sm}(l^{sm},t) = Q_{mes}^m(t), & s \in I_m^-,\ if\ I_m^+ = \varnothing, \end{cases} \quad m \in I_q^f, (7)$$

When the spots of oil leakages from a pipeline and the rates of these leakages are known $\xi_{ks}, q_{ks}^{loss}(t)$, $(k,s) \in J$, it is sufficient to use one of the boundary-value conditions (6) or (7) to calculate the regime of liquid motion in the pipeline from (1) on the time interval $[t_0,T]$. One of them we will use in the functional, the form which will be given below.

The problem consists in the detection of the points of leakage $\xi = \{\xi_{ks},(k,s) \in J\}$ and corresponding losses of raw material $q^{loss}(t) = \{q_{ks}^{loss}(t),(ks) \in J\}$ at $t \in [t_0,T]$ with the use of the given mathematical model and obtained information.

It is important to note that if process (1) is rather long, then, due to the presence of friction typical of any real physical system, the influence of the initial state of the pipeline on the regimes of oil motion in it becomes weaker with time. Therefore, when the process is observed for a long time, i.e., within a large time interval $[t_0, T]$, the influence of the initial regime of oil flow in a pipeline (at $t = t_0$) on the current state of the process decreases, and there exists such τ $(\tau < T)$ that at $t > \tau$ the regime of oil motion experiences only the influence of the boundary-value conditions on the time interval $[t_0, T]$, where the quantity τ is determined by the parameters of the process and the characteristics of the pipeline [9].

Therefore, we will assume that at the initial moment of time t_0 the initial conditions for the process (36) are not exactly known, and some sets of possible values of the initial modes are specified, which are defined in this case by a parametric set $D \subset R^{M+n}$ of possible values of flow rates in the sections under steady-state flow modes:

$$\begin{aligned} &\hat{Q}_\gamma^{ks}(x) = Q^{ks}(x, t_0; \gamma) = \gamma_q^{ks} = const, \\ &\hat{P}_\gamma^{ks}(x) = P^{ks}(x, t_0; \gamma) = \gamma_p^{ks} - 2ax\gamma_q^{ks}, \\ &x \in (0, l^{ks}), \ (k,s) \in J, \\ &\gamma = (\gamma_p, \gamma_q) = (\gamma_p^{ks}, \gamma_q^{ks})_{\substack{ks \in J \\ k \in I}} \in D \subset R^{M+n}. \end{aligned} \quad (8)$$

Here γ_q^{ks} – possible value of flow rates in the (k,s)–th section $ks \in J$, γ_p^{ks} – possible value of pressures at the nodes $k \in I$ under steady-state flow modes, the corresponding density functions are given, which are written in vector form as $\mu_D(\gamma)$.

Possible set of initial states can also be determined as by finite set of their values, as well as by the set of parametrically given functions:

$$\{\hat{Q}_{\gamma_1}^{ks}(x), \hat{Q}_{\gamma_2}^{ks}(x), ..., \hat{Q}_{\gamma_N}^{ks}(x)\}, \{\hat{P}_{\gamma_1}^{ks}(x), \hat{P}_{\gamma_2}^{ks}(x), ..., \hat{P}_{\gamma_N}^{ks}(x)\},$$

In order to solve the problem posed, we will consider the functional that determines the derivation of regimes of oil flow at the given points of the oil pipeline section from those predicted:

$$\Im(\xi, q^{loss}) = \int_D \left[\Phi(\xi, q^{loss}; \gamma) + \Re(\xi, q)\right] \mu_D(\gamma) d\gamma \to \min, \quad (9)$$

$$\Phi(\xi, q^{loss}; \gamma) = \sum_{m \in \tilde{I}_q^f} \int_\tau^T [Q^m(t; \xi, q(t), \gamma) - Q_{mes}^m(t)]^2 dt, \quad (10)$$

$$\Re(\xi, q) = \varepsilon_1 \left\| q(t) - \hat{q} \right\|_{L_2^Z[t_0, T]}^2 + \varepsilon_2 \left\| \xi - \hat{\xi} \right\|_{R^Z}^2, \quad (11)$$

where $Q^m(t; \xi, q(t), \gamma), m \in \tilde{I}_q^f$ – is the solution of the problem (1)–(5),(7),(8),(10) at the given values of $(\xi, q^{loss}(t))$, $[\tau, T]$ is the time interval of monitoring the process whose regimes already do not depend on the initial conditions; $\tilde{\xi}, \tilde{q} \in R^m, \varepsilon_1, \varepsilon_2$ – are the regularization parameters. Since the initial conditions at time t_0 do not influence the process in the interval $[\tau, T]$, exact knowledge of the initial value of t_0 is not of primary importance.

Proceeding from the meaning of the problem considered, technological conditions, and technical requirements, we will assume that are restrictions on the identified functions and parameters:

$$0 < \xi_{ks} \le l^{ks}, \quad \underline{q} \le q^{loss}(t) \le \overline{q}, \quad t \in [t_0, T],$$

where $\underline{q}, \overline{q}$ are the given quantities.

As is seen, as to the determination of the points and rates of leakages the posed problem is the problem of parametric optimal control of an object described by a hyperbolic system. For its solution we use numerical methods (projections of the conjugated gradient) based on iteration procedures of first order optimization. To carry out this procedure, it is necessary to obtain formulas for the gradient of functional. If as a result of the solution of posed problem we obtain that $\left|q^{loss}(t)\right| \le \varepsilon, \ t \in [\tau, T]$, this will mean that in this section of the pipeline network there is no leakage of raw material.

III. Numerical Solution to the Problem

Theorem. In problem (1) - (11), the components of the gradient of functional (9) with respect to admissible places and volumes of leakages $(\xi^{\bar{k}\bar{s}}, q^{\bar{k}\bar{s}}(t)), \ (\bar{k}, \bar{s}) \in J^{loss}$ are determined by the formulas:

$$grad_{q^{\bar{k}\bar{s}}} \Im(\xi, q) = \int_D \left\{ grad_{q^{\bar{k}\bar{s}}} \Phi(\xi, q; \gamma) + 2\varepsilon(q^{\bar{k}\bar{s}}(t) - \tilde{q}^{\bar{k}\bar{s}}(t)) \right\} \mu_D(\gamma) d\gamma,$$

$$grad_{\xi^{\bar{k}\bar{s}}} \Im(\xi, q) = \int_D \left\{ grad_{\xi^{\bar{k}\bar{s}}} \Phi(\xi, q; \gamma) + 2\varepsilon(\xi^{\bar{k}\bar{s}} - \hat{\xi}^{\bar{k}\bar{s}}) \right\} \mu_D(\gamma) d\gamma,$$

$$grad_{q^{\bar{k}\bar{s}}} \Im(\xi, q) =$$

$$= \int_D \left\{ c^2 \frac{\rho}{S^{\bar{k}\bar{s}}} \psi^{\bar{k}\bar{s}}(\xi^{\bar{k}\bar{s}}, t; \gamma) + 2\varepsilon(q^{\bar{k}\bar{s}}(t) - \tilde{q}^{\bar{k}\bar{s}}(t)) \right\} \mu_D(\gamma) d\gamma,$$

$$t \in [t_0, T],$$

$$grad_{\xi^{\bar{k}\bar{s}}} \Im(\xi, q(t)) =$$

$$\int_D \left\{ c^2 \frac{\rho}{S^{\bar{k}\bar{s}}} \int_{t_0}^T q^{\bar{k}\bar{s}}(t) (\psi^{\bar{k}\bar{s}}(x,t))'_x \big|_{x=\xi^{\bar{k}\bar{s}}} dt + 2\varepsilon_2(\xi^{\bar{k}\bar{s}} - \hat{\xi}^{\bar{k}\bar{s}}) \right\} \mu_D(\gamma) d\gamma$$

Here the functions $\psi^{ks}(x,t) = \psi^{ks}(x,t;\gamma), \ (k,s) \in J$ are the solutions of the conjugate initial-boundary value problem with nonlocal boundary conditions, corresponding to the direct problem.

Let the functions $\phi^{ks}(x,t)$, $\psi^{ks}(x,t)$, $s \in I_k^+, k \in I$ are the solutions to the next adjoint boundary value problem:

$$\begin{aligned} &-\frac{\partial \phi^{ks}(x,t)}{\partial x} = \frac{\partial \psi^{ks}(x,t)}{\partial t}, \\ &-\frac{\partial \phi^{ks}(x,t)}{\partial t} = c^2 \frac{\partial \psi^{ks}(x,t)}{\partial x} - 2a^{ks}\phi^{ks}(x,t), \\ &t \in (0,T], \ x \in (0, l^{ks}), s \in I_k^+, k \in I. \end{aligned} \quad (12)$$

$$\phi^{ks}(x,T) = 0, \ \psi^{ks}(x,T) = 0, \ x \in [0, l^{ks}], \ s \in I_k^+, k \in I,$$

$$\psi(l^m, t) = -2\frac{S^{ks}}{\rho}[Q^m(t; \xi, q^{loss}) - Q_{mes}^m(t)],$$

$$s \in I_m^-, \ if \ I_m^+ = \varnothing, \ m \in I_q^f,$$

$$\psi(0,t) = -2\frac{S^{ks}}{\rho}[Q^m(t;\xi,q^{loss}) - Q^m_{mes}(t)],\ s \in I^+_m,\ if\ I^-_m = \varnothing,$$

$$m \in I^f_q,\quad \sum_{s\in I^+_k}\phi^{ks}(l^{ks},t) - \sum_{s\in I^-_k}\phi^{ks}(0,t) = 0,\ k \in I$$

$$\psi^{k_ik}(l^{k_ik},t) = \psi^{kk_j}(0,t),\ k_i \in I^+_k, k_j \in I^-_k, k \in I.$$

IV. The results of numerical experiments

We will consider the following specially constructed test problem for oil pipeline network consisting of 5 nodes, as shown in figure 3. Here

$N = 6$, $M = 5, I^f\{1,3,4,6\}, N_f = 4, N_{\text{int}} = 2$.

There are no external inflows and outflows inside the network.

We assume that in the course of 30 min we observe the process (mode of operation of pumping plants at the ends of the sections) of oil transportation with the kinematic viscosity $\nu = 1.5\cdot 10^{-4}\,(m^2/s)$ and density $\rho = 920\,(kg/m^3)$ ($2a = 0.017$ for case being considered; the sound velocity in oil is $1200\,(m/s)$) in the sections of pipeline of diameter 530 (mm), of the lengths of the segments:

$l^{(1,2)} = 100$ (км), $l^{(5,2)} = 30$ (км), $l^{(3,2)} = 70$ (км),

$l^{(5,4)} = 100$ (км), $l^{(5,6)} = 60$ (км).

Let there was regime in the pipes at initial time instance $t = 0$ with the following values of pressure and flow rate in the pipes:

$\hat{P}^{1,2}(x) = 2300000\text{-}5.8955x$ (Pa),

$\hat{P}^{5,2}(x) = 1745669\text{-}1.17393x$ (Pa),

$\hat{P}^{3,2}(x) = 1827844\text{-}1.677043x$ (Pa),

$\hat{P}^{5,4}(x) = 1827844\text{-}2.35786x$ (Pa),

$\hat{P}^{5,6}(x) = 1827844\text{-}0.94415x$ (Pa). (17)

$\hat{Q}^{1,2}(x) = 300\ (m^3/hour)$,

$\hat{Q}^{5,2}(x) = 200\ (m^3/hour)$ $\hat{Q}^{3,2}(x) = 100\ (m^3/hour)$,

$\hat{Q}^{5,4}(x) = 120\ (m^3/hour)$, $\hat{Q}^{5,6}(x) = 80\ (m^3/hour)$, (18)

Let the oil flow rate at the ends of this pipeline section be defined by the functions:

$\tilde{P}^1_0(t) = 2000000\text{+}300000\,e^{-0.0003t}$ (Pa),

$\tilde{P}^3_0(t) = 1900000\text{-}72156\,e^{-0.0004t}$ (Pa), (19)

$\tilde{P}^4_l(t) = 1800000\text{-}66571\,e^{-0.0007t}$ (Pa),

$\tilde{P}^6_l(t) = 1600000\text{+}86372\,e^{-0.0002t}$ (Pa). (20)

On the assumption that the point of leakage is located at the point $\xi = 30\ (km)$ of the first section of pipeline network and the rate of leakage is determined by the function $q^{loss}(t) = 50 - 10e^{-0.0003t}\ (m^3/h)$, we solved the boundary-value problem (1)-(6) numerically and determined the numerical values of pressure at the ends of the section $P^n(t), n \in I^f_p$. Thereafter, with the aid of the probe of uniformly distributed random numbers these values were changed within _2% (to simulate the error of measurements) and used as the observed regimes of the process. The point and rate of leakage $\xi,\ q^{loss}(t)$ "forgotten" in this case.

It is required to determine $\xi,\ q^{loss}(t)$ with the aid of the above-suggested method of solving problem (1)–(5),(7),(8),(10). For this purpose, we used the method of the projection of conjugate gradients. The numerical solution of the boundary-value problem (1)–(5),(7) was made using the scheme of the sweep method [7], on the grids with the pitches $h_x = 10$м and $h_t = 100\,(сек)$.

Table 1 presents the obtained results of the minimization of functional (10) for different initial values of the identified parameters $(\xi,\ q^{loss}(t))^0$, as well as the required number of iterations (one-dimensional minimizations) of the method of projection of conjugate gradients.

Table1. The results of experiments ($\alpha = e^{-0.0003t}$)

ξ_0 (km)	$q_0^{loss}(t)$ $(m^3/hour)$	ξ_* (km)	$\Im_0$	$\Im_*$	Number of iter.
60,00	$90-10\alpha$	30,00	76,10	$5{,}73\cdot 10^{-7}$	6
20,00	$20-10\alpha$	29,99	16,70	$1{,}26\cdot 10^{-7}$	5
90,00	$30-10\alpha$	30,00	11,66	$3{,}19\cdot 10^{-6}$	16
10,00	$66+20\alpha$	29,99	42,48	$1{,}85\cdot 10^{-6}$	14
45,68	$66+20\alpha$	29,99	57,63	$7{,}43\cdot 10^{-7}$	8

V. Conclusion

A numerical approach to determining leak locations and amounts in an unsteady fluid flow in a pipeline network of complex structure was proposed. The mathematical formulation of the considered problem was reduced to the class of parametric optimal control problems for a system of a large number of hyperbolic partial differential equations. A feature that causes special difficulties in the numerical solution is that the problem involves nonseparated boundary conditions at network nodes, which appear due to an analogue of Kirchhoff's law concerning material balance conservation and the continuity of the pressure value. Another specific feature is that no initial conditions are specified, since the process is too long; additionally, impulse functions are involved in the differential equations. Formulas for the gradient of the cost functional with respect to the identification parameters were obtained, which make it possible to use efficient numerical optimization methods of the first order.

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